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Transition rates and radiative lifetimes of Ca I

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ARTICLE INFO

Article history:

Received 9 February 2017

Received in revised form 20 February 2017

Accepted 28 March 2017

Available online 26 April 2017

Keywords:

Atomic data

Transition probabilities

Lifetime

ABSTRACT

We tabulate spontaneous emission rates for all possible 811 electric-dipole-allowed transitions between the 75 lowest-energy states of Ca I. These involve the 4sns ($n = 4\text{--}8$), 4snp ($n = 4\text{--}7$), 4snd ($n = 3\text{--}6$), 4snf ($n = 4\text{--}6$), 3d², 4p², 3d4p, and 4s5g electronic configurations. We compile the transition rates by carrying out *ab initio* relativistic calculations using the combined method of configuration interaction and many-body perturbation theory. The results are compared to the available literature values. The tabulated rates can be useful in various applications, such as optimizing laser cooling in magneto-optical traps, estimating various systematic effects in optical clocks and evaluating static or dynamic polarizabilities and long-range atom–atom interaction coefficients and related atomic properties.

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1. Introduction

Alkaline-earth atoms and divalent-like atoms (such as Yb and Hg) became a subject of interest to the cold atom community in the past decade. These atoms possess two valence electrons outside a tightly bound core and in the LS coupling scheme, the atomic states can be classified by being either singlet or triplet states. The availability of relatively wide spin-allowed and narrow spin-forbidden electric-dipole (E1) transitions enables stacking laser cooling on both types of transitions, with the spin-allowed transitions used for the initial rapid cooling and spin-forbidden transitions—for reaching much lower Doppler-limit temperatures. Moreover, the narrow inter-combination transitions, such as the 4s² 1S₀ – 4s4p 3P₁ transition, can be used as an optical frequency Refs. [1,2]. The highly-forbidden 4s² 1S₀ – 4s4p 3P₀ transition can be potentially used for an optical lattice clock scheme [3]. However, due to peculiarities of its electronic structure, Ca, unlike other alkaline-earth atoms, has a relatively short lifetime in conventional magneto-optical traps (MOT). To improve the MOT efficiency, several re-pumping schemes were proposed and demonstrated [4]. That work required reliable electric-dipole transition data for many transitions between the 75 lowest energy states of neutral Ca. Here, we compile the results of our computational work that served as a basis of the MOT performance analysis [4]. We anticipate that the tabulated data will be useful in multiple other contexts, such as estimating various systematic effects in optical clocks and computing static or dynamic polarizabilities and long-range inter-atomic interaction coefficients.

There have been a number of atomic-structure calculations for neutral calcium. The earlier work includes multi-configuration Hartree–Fock (MCHF) calculation [5] and semi-empirical model-potential calculations [6–9]. These computations provide oscillator strengths for spin-allowed transitions for levels up to 4s10s, 4s9p and 4s6d. Most of them are non-relativistic with very limited numbers of low-lying levels treated with *ab initio* relativistic methods. In particular, Fisher and Tachiev [10] reported energies and E1 transition rates for levels below 3d4p 1F₃. Porsev et al. [11] and Savukov and Johnson [12] computed the 4s² – 4s4p, 4s4p – 3d4s, and 4s4p – 4s5s transition rates using a combination of configuration-interaction (CI) and many-body perturbation theory (MBPT) (referred to as the CI+MBPT method). The CI+MBPT method results were in excellent agreement with high-precision experimental values. This fact partially motivated our use of the relativistic CI+MBPT method for the present work.

2. Computational details

The CI+MBPT method employs a systematic formalism that combines advantages of both configuration interaction (CI)

method and many-body perturbation theory (MBPT) [13]. This method has been used extensively for evaluation of atomic properties (see, e.g., review [14] for optical lattice clock applications and references therein). Relativistic effects are included exactly as the formalism starts from the Dirac equation and employs relativistic bi-spinor wave functions throughout the entire calculation. In our treatment, the CI model space is limited to excitations of the two valence electrons. Contributions involving virtual excitations of core orbitals are treated within the MBPT. In this approach, we first solve for the valence electron orbitals and energies in the field of core electrons. The one-electron effective potential includes both the frozen-cored Dirac–Hartree–Fock (DHF V^{N-2}) and self-energy (core-polarization) potentials. The self-energy correction is computed using second-order MBPT diagrams involving virtual core excitations. At the next step, the computed one-electron valence orbitals are used to diagonalize the atomic Hamiltonian in the model space of two valence electrons within the CI method. The CI model-space Hamiltonian includes the residual (beyond DHF) Coulomb interaction between the valence electrons and also their core-polarization-mediated interaction. The latter was computed in the second-order MBPT. This step yields two-electron wavefunctions and energies. Finally, with the obtained wave-functions we calculated the required electric-dipole matrix elements. In calculations of transition rates we used experimental energy intervals and the computed CI+MBPT matrix elements.

We used two independent CI+MBPT implementations: (i) by the Reno group (see the description of the earlier version in Ref. [15]) and (ii) the recently published package [16]. The practical goal of the calculations was not reaching the best possible accuracy, but rather the generation of massive amounts of reliable data for the transition array involving 75 lowest-energy levels. The Reno code was run on a large basis set but without including core-polarization-mediated interaction in the CI Hamiltonian due to considerable computational costs. The production runs with package [16] employed a smaller basis set (due to code limitations) but treated the correlation problem more fully.

While using the package [16] we employed the one-electron basis set that included the 1s–17s, 2p–17p, 3d–17d, 4f–17f, and 5g–17g orbitals, where the core and 4s, ..., 6f orbitals are DHF ones, while the remaining orbitals were represented by a B-spline basis set. The Reno code used the dual-kinetic-balance basis set generated in the DHF V^{N-2} potential using spherical cavity of 75 Bohr radius [17]. The basis included orbitals with orbital angular momentum ℓ up to 6. The total number of positive-energy (in the Dirac sense) orbitals per partial wave was 40 with the 35 lowest-energy orbitals used in the calculations.

For most states, the values of transition rates obtained with package [16] were in close agreement with the NIST recommended

Explanation of Tables

In Table 1, we compile radiative E1 lifetimes of the 74 lowest-energy excited states of Ca I. In Tables 2–6, we tabulate our theoretical data for spontaneous emission rates of all possible 811 E1-allowed transitions between the 75 lowest-energy states of Ca I. A comparison with other literature values is presented in Table 7.

Table 1. Radiative electric-dipole decay rates A_{total} and lifetimes τ of Ca I.

First column	States.
Second and third columns	Energies in cm^{-1} , our CI+MBPT and the NIST values.
Fourth column	Difference between our CI+MBPT and NIST energies in %.
Fifth column	Total spontaneous emission rates A_{total} in 10^8 s^{-1} for the state.
Sixth and seventh columns	Lifetime of the state, our and the literature values.
MPCI	Model Potential Configuration Interaction method.
MCHF	Multi-Configuration Hartree–Fock method.
Exp.	Experimental values.

Table 2. Spontaneous emission rates of transitions originating from the $4sns$ ($n = 4–8$) states of Ca I.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Transition wavelength λ , in nm.
Fourth and Fifth columns	Our values of spontaneous emission rates A_{if} in 10^8 s^{-1} in length (L) and velocity (V) forms of the E1 operator.
Sixth column	Difference between the L and V column values defined as $(L - V)/L$ in %.
Seventh column	Branching ratio (BR) in %, defined as A_{if}/A_{total} .

Table 3. Spontaneous emission rates of transitions originating from the $4snp$ ($n = 4–7$) states of Ca I.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Transition wavelength λ , in nm.
Fourth and Fifth columns	Our values of spontaneous emission rates A_{if} in 10^8 s^{-1} in length (L) and velocity (V) forms of the E1 operator.
Sixth column	Difference between the L and V column values defined as $(L - V)/L$ in %.
Seventh column	Branching ratio (BR) in %, defined as A_{if}/A_{total} .

Table 4. Spontaneous emission rates of transitions originating from the $4snd$ ($n = 3–6$) states of Ca I.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Transition wavelength λ , in nm.
Fourth and Fifth columns	Our values of spontaneous emission rates A_{if} in 10^8 s^{-1} in length (L) and velocity (V) forms of the E1 operator.
Sixth column	Difference between the L and V column values defined as $(L - V)/L$ in %.
Seventh column	Branching ratio (BR) in %, defined as A_{if}/A_{total} .

Table 5. Spontaneous emission rates of transitions originating from the $4snf$ ($n = 4–6$) states of Ca I.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Transition wavelength λ , in nm.
Fourth and Fifth columns	Our values of spontaneous emission rates A_{if} in 10^8 s^{-1} in length (L) and velocity (V) forms of the E1 operator.
Sixth column	Difference between the L and V column values defined as $(L - V)/L$ in %.
Seventh column	Branching ratio (BR) in %, defined as A_{if}/A_{total} .

Table 6. Spontaneous emission rates of transitions originating from the $3d^2$, $4p^2$, $3d4p$, and $4s5g$ states of Ca I.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Transition wavelength λ , in nm.
Fourth and Fifth columns	Our values of spontaneous emission rates A_{if} in 10^8 s^{-1} in length (L) and velocity (V) forms of the E1 operator.
Sixth column	Difference between the L and V column values defined as $(L - V)/L$ in %.
Seventh column	Branching ratio (BR) in %, defined as A_{if}/A_{total} .

Table 7. Comparison of transition probabilities with literature values.

First and Second columns	Transition levels, the upper (initial) and the lower (final) level.
Third column	Spontaneous emission rates A_{if} in units of 10^8 s^{-1} using the L -form of the E1 operator.
Forth and Fifth columns	Other theoretical A_{if} values and the % difference between this work and other theoretical data, calculated as the difference of the third and forth column values divided by the third column value.
Sixth and Seventh columns	NIST recommended A_{if} value and the % difference between this work and NIST data, computed as the difference between the third and sixth column values divided by the third column value.
L	Length form of the electric-dipole operator.
MPCI	Model Potential Configuration Interaction method.
MCHF	Multi-Configuration Hartree–Fock method.
CI+MBPT	Configuration Interaction with Many-Body Perturbation Theory method.
NIST	NIST recommended values.

Table 1 (continued)

State	Energy (cm^{-1})		Diff. (%)	$A_{\text{total}}(10^8 \text{ s}^{-1})$	τ (ns)	
	This work	NIST			This work	Refs.
$4s6d\ ^3D_2$	45893.340	45050.420	1.9	0.377	26.5	
$4s6d\ ^3D_3$	45897.080	45052.370	1.9	0.376	26.6	
$4s7p\ ^1P_1$	46975.130	45425.360	3.4	0.407	24.5	
$4s8s\ ^3S_1$		45738.680		0.009	105	149 MPCI [9] 118.3(6.3) Exp. [25]
$4s8s\ ^1S_0$		45887.200		0.006	157	
$4s6f\ ^3F_2$	47711.820	46164.640	3.4	0.227	112.7	
$4s6f\ ^3F_3$	47712.420	46164.790	3.4	0.228	113.7	
$4s6f\ ^3F_4$	47713.200	46164.970	3.4	0.228	121.9	
$4s6f\ ^1F_3$	47768.330	46182.400	3.4	0.291	96.7	

Table 2 (continued)

Transition		λ (nm)	$A_{if} (10^8 \text{ s}^{-1})$	Diff. (%)	BR. (%)
Initial	Final		<i>L</i>	<i>V</i>	
4s8s 3S_1	4s6p 1P_1	2463.254	2.019E–08	0.00	
4s8s 3S_1	4s4f 3F_2	2802.321	8.155E–13	0.00	
4s8s 3S_1	4s4p 3P_2	328.701	3.693E–02	38.96	
4s8s 3S_1	4s6p 3P_0	3101.901	8.286E–04	0.87	
4s8s 3S_1	4s6p 3P_1	3105.619	2.469E–03	2.61	
4s8s 3S_1	4s6p 3P_2	3113.238	4.063E–03	4.29	
4s8s 3S_1	4snp 1P_1	5539.552	1.078E–08	0.00	
4s8s 3S_1	4s5f 3F_2	10245.272	2.432E–13	0.00	
4s8s 3S_1	4s7p 3P_0	12771.229	7.410E–04	0.78	
4s8s 3S_1	4s7p 3P_1	12803.769	2.215E–03	2.34	
4s8s 3S_1	4s7p 3P_2	12871.338	3.664E–03	3.87	
4s8s 3S_1	4s7p 1P_1	31916.252	2.029E–08	0.00	
4s8s 3S_1	4s4p 1P_1	452.768	2.558E–07	0.00	
4s8s 1S_0	4s5p 3P_1	1071.530	1.821E–04	0.29	
4s8s 1S_0	4s5p 1P_1	1092.230	1.964E–02	30.78	
4s8s 1S_0	3d4p 3D_1	1299.577	9.657E–06	0.02	
4s8s 1S_0	3d4p 3P_1	1526.279	7.658E–07	0.00	
4s8s 1S_0	4s4p 3P_1	325.976	6.115E–08	0.00	
4s8s 1S_0	4s6p 1P_1	2376.319	6.149E–03	9.63	
4s8s 1S_0	4s6p 3P_1	2968.689	1.757E–07	0.00	
4s8s 1S_0	4snp 1P_1	5118.441	7.344E–05	0.12	
4s8s 1S_0	4s7p 3P_1	10758.009	5.245E–07	0.00	
4s8s 1S_0	4s7p 1P_1	21652.520	2.071E–03	3.25	
4s8s 1S_0	4s4p 1P_1	449.743	3.569E–02	55.93	

Table 3 (continued)

Transition		λ (nm)	$A_{if}(10^8 \text{ s}^{-1})$		Diff. (%)	BR. (%)
Initial	Final		<i>L</i>	<i>V</i>		
4s7p 3P_2	3d4s 1D_2	432.673	7.948E−08	2.350E−07	−195.65	0.00
4s7p 1P_1	4s5s 3S_1	720.157	1.497E−06	1.062E−06	29.07	0.00
4s7p 1P_1	4s 2 1S_0	220.141	1.295E−01			55.13
4s7p 1P_1	4s5s 1S_0	825.893	1.135E−02	1.209E−02	−6.49	2.79
4s7p 1P_1	4s4d 1D_2	1230.456	4.083E−03	5.034E−03	−23.28	1.00
4s7p 1P_1	4s4d 3D_1	1302.565	1.678E−09	1.352E−08	−705.49	0.00
4s7p 1P_1	4s4d 3D_2	1303.188	2.058E−07	2.067E−07	−0.45	0.00
4s7p 1P_1	4p 2 3P_0	1426.977	5.766E−07	4.382E−07	24.01	0.00
4s7p 1P_1	4p 2 3P_1	1436.668	3.708E−08	1.279E−07	−245.02	0.00
4s7p 1P_1	4p 2 3P_2	1454.799	2.469E−07	1.081E−06	−337.68	0.00
4s7p 1P_1	4s6s 3S_1	2019.745	7.975E−07	6.883E−07	13.69	0.00
4s7p 1P_1	4s6s 1S_0	2111.968	4.763E−03	4.261E−03	10.53	1.17
4s7p 1P_1	4p 2 1D_2	2125.168	1.349E−03	1.870E−03	−38.60	0.33
4s7p 1P_1	4p 2 1S_0	2747.947	7.203E−07	5.423E−05	−7429.71	0.00
4s7p 1P_1	4s5d 3D_1	3728.060	5.631E−09	4.271E−10	92.42	0.00
4s7p 1P_1	4s5d 3D_2	3730.452	1.906E−10	3.758E−09	−1872.01	0.00
4s7p 1P_1	4s5d 1D_2	3989.929	7.689E−04	1.177E−04	84.70	0.19
4s7p 1P_1	3d 2 3F_2	5126.812	3.402E−07	1.639E−07	51.81	0.00
4s7p 1P_1	3d4s 3D_1	398.565	1.918E−09	3.078E−08	−1504.30	0.00
4s7p 1P_1	4s7s 3S_1	6922.379	2.786E−07	5.027E−07	−80.46	0.00
4s7p 1P_1	4s7s 1S_0	8704.584	2.597E−03	4.065E−03	−56.57	0.64
4s7p 1P_1	3d4s 3D_2	398.786	1.169E−05	1.428E−05	−22.12	0.00
4s7p 1P_1	4s6d 1D_2	22960.531	4.250E−04	1.325E−04	68.83	0.10
4s7p 1P_1	4s6d 3D_1	26575.248	6.286E−09	3.545E−09	43.61	0.00
4s7p 1P_1	4s6d 3D_2	26670.934	6.608E−07	1.930E−07	70.80	0.00
4s7p 1P_1	3d4s 1D_2	424.165	1.575E−01	1.942E−01	−23.35	38.64

Table 4 (continued)

Transition		λ (nm)	$A_{if}(10^8 \text{ s}^{-1})$		Diff. (%)	BR. (%)
Initial	Final		L	V		
$4s6d\ ^3D_3$	$4s6p\ ^3P_2$	3959.173	1.005E−02	1.353E−02	−34.57	2.68
$4s6d\ ^3D_3$	$4s5f\ ^3F_2$	34512.511	2.834E−07	1.443E−07	49.09	0.00
$4s6d\ ^3D_3$	$4s5f\ ^3F_3$	34538.735	9.918E−06	5.048E−06	49.10	0.00
$4s6d\ ^3D_3$	$4s5f\ ^3F_4$	34572.169	1.145E−04	5.826E−05	49.11	0.03
$4s6d\ ^3D_3$	$4s5f\ ^1F_3$	40405.673	1.512E−12	1.608E−13	89.36	0.00
$4s6d\ ^3D_3$	$4s7p\ ^3P_2$	110363.095	3.748E−06	6.449E−07	82.79	0.00

Table 5 (continued)

Transition		λ (nm)	$A_{if}(10^8 \text{ s}^{-1})$	Diff. (%)	BR. (%)
Initial	Final		<i>L</i>	<i>V</i>	
4s6f 1F_3	4s5d 3D_3	2911.200	7.052E−09	0.00	
4s6f 1F_3	4s5d 1D_2	3064.336	5.037E−03	4.87	
4s6f 1F_3	3d 2 3F_2	3693.349	2.889E−05	0.03	
4s6f 1F_3	3d 2 3F_3	3712.945	3.051E−10	0.00	
4s6f 1F_3	3d 2 3F_4	3739.282	1.498E−09	0.00	
4s6f 1F_3	4s5g 1G_4	387.100	7.996E−05	0.08	
4s6f 1F_3	4s6d 1D_2	7654.330	5.901E−03	5.71	
4s6f 1F_3	4s6d 3D_2	8385.252	6.309E−06	0.01	
4s6f 1F_3	4s6d 3D_3	8834.078	6.540E−11	0.00	
4s6f 1F_3	3d4s 3D_2	8849.323	9.069E−05	0.09	
4s6f 1F_3	3d4s 3D_3	387.426	7.248E−08	0.00	
4s6f 1F_3	3d4s 1D_2	410.968	8.390E−02	81.13	

Table 6 (continued)

Transition		λ (nm)	$A_{if}(10^8 \text{ s}^{-1})$	Diff. (%)	BR. (%)
Initial	Final		L	V	
$3d^2 \ ^3F_2$	$4s4f \ ^1F_3$	8839.857	3.297E-08	2.348E-08	28.79
$3d^2 \ ^3F_2$	$4s6p \ ^3P_1$	10458.938	1.945E-10	1.992E-10	-2.41
$3d^2 \ ^3F_2$	$4s6p \ ^3P_2$	10545.853	5.590E-11	6.354E-11	-13.65
$3d^2 \ ^3F_2$	$4s4p \ ^1P_1$	504.476	3.937E-05	4.524E-05	-14.90
$3d^2 \ ^3F_3$	$3d4p \ ^3F_2$	1288.881	2.306E-03	2.934E-03	-27.24
$3d^2 \ ^3F_3$	$3d4p \ ^3F_3$	1303.711	3.596E-02	4.524E-02	-25.80
$3d^2 \ ^3F_3$	$3d4p \ ^1D_2$	1306.556	9.782E-04	1.232E-03	-25.90
$3d^2 \ ^3F_3$	$3d4p \ ^3F_4$	1317.136	3.292E-03	4.159E-03	-26.33
$3d^2 \ ^3F_3$	$4s5p \ ^3P_2$	1446.341	1.068E-06	1.125E-06	-5.35
$3d^2 \ ^3F_3$	$3d4p \ ^3D_2$	1897.533	2.249E-02	2.136E-02	5.02
$3d^2 \ ^3F_3$	$3d4p \ ^3D_3$	1912.046	2.605E-03	2.454E-03	5.80
$3d^2 \ ^3F_3$	$3d4p \ ^3P_2$	2410.196	3.913E-06	3.858E-06	1.40
$3d^2 \ ^3F_3$	$3d4p \ ^1F_3$	3388.418	1.948E-07	1.508E-07	22.57
$3d^2 \ ^3F_3$	$4s4f \ ^3F_2$	7582.018	1.001E-08	6.587E-08	-558.24
$3d^2 \ ^3F_3$	$4s4f \ ^3F_3$	7584.031	1.028E-07	6.809E-07	-562.43
$3d^2 \ ^3F_3$	$4s4p \ ^3P_2$	354.948	1.583E-07	6.037E-08	61.86
$3d^2 \ ^3F_3$	$4s4f \ ^3F_4$	7586.735	9.143E-09	6.170E-08	-574.86
$3d^2 \ ^3F_3$	$4s4f \ ^1F_3$	8729.584	1.863E-09	4.751E-10	74.50
$3d^2 \ ^3F_3$	$4s6p \ ^3P_2$	10389.287	3.400E-10	3.393E-10	0.19
$3d^2 \ ^3F_4$	$3d4p \ ^3F_3$	1300.495	2.489E-03	3.174E-03	-27.54
$3d^2 \ ^3F_4$	$3d4p \ ^3F_4$	1313.853	3.893E-02	4.913E-02	-26.20
$3d^2 \ ^3F_4$	$3d4p \ ^3D_3$	1905.136	2.499E-02	2.372E-02	5.08
$3d^2 \ ^3F_4$	$3d4p \ ^1F_3$	3366.777	7.887E-08	8.891E-08	-12.74
$3d^2 \ ^3F_4$	$4s4f \ ^3F_3$	7476.468	8.641E-09	5.454E-08	-531.16
$3d^2 \ ^3F_4$	$4s4f \ ^3F_4$	7479.096	1.246E-07	7.938E-07	-537.00
$3d^2 \ ^3F_4$	$4s4f \ ^1F_3$	8587.377	1.090E-10	4.795E-10	-339.84
$4s5g \ ^3G_5$	$3d4p \ ^3F_4$	1113.838	5.774E-04	5.741E-04	0.57
$4s5g \ ^3G_5$	$4s4f \ ^3F_4$	3698.457	5.062E-02	4.824E-02	4.70
$4s5g \ ^3G_5$	$4s5f \ ^3F_4$	89493.467	4.431E-06	4.434E-06	-0.07
$4s5g \ ^1G_4$	$3d4p \ ^3F_3$	1104.089	4.806E-05	4.773E-05	0.69
$4s5g \ ^1G_4$	$3d4p \ ^3F_4$	1113.702	2.859E-06	2.823E-06	1.24
$4s5g \ ^1G_4$	$3d4p \ ^3D_3$	1511.298	7.408E-07	7.175E-07	3.14
$4s5g \ ^1G_4$	$3d4p \ ^1F_3$	2305.178	2.755E-02	2.658E-02	3.53
$4s5g \ ^1G_4$	$4s4f \ ^3F_3$	3696.325	3.928E-03	3.743E-03	4.69
$4s5g \ ^1G_4$	$4s4f \ ^3F_4$	3696.967	2.649E-04	2.523E-04	4.77
$4s5g \ ^1G_4$	$4s4f \ ^1F_3$	3948.886	3.606E-02	3.409E-02	5.45
$4s5g \ ^1G_4$	$4s5f \ ^3F_3$	88409.513	3.588E-07	3.594E-07	-0.17
$4s5g \ ^1G_4$	$4s5f \ ^3F_4$	88628.911	2.384E-08	2.424E-08	-1.68
$4s5g \ ^1G_4$	$4s5f \ ^1F_3$	140706.346	9.701E-07	2.417E-05	-2391.78

Table 7 (continued)

Final state	Initial state	This work ($L, 10^8 \text{ s}^{-1}$)	Other theory (10^8 s^{-1})	Diff.	NIST	Diff. (%)
$4s5f\ ^3F_2$	$3d4s\ ^3D_3$	5.884E–04			5.8E–04	1.45
$4s5f\ ^3F_3$	$3d4s\ ^3D_2$	1.172E–01			1.2E–01	–2.36
$4s5f\ ^3F_3$	$3d4s\ ^3D_3$	1.471E–02			1.5E–02	–1.97
$4s5f\ ^3F_4$	$3d4s\ ^3D_3$	1.323E–01			1.3E–01	1.79
$4s7s\ ^3S_1$	$4s4p\ ^3P_0$	1.873E–02			1.3E–02	44.07
$4s7s\ ^3S_1$	$4s4p\ ^3P_1$	5.630E–02			4.6E–02	22.38
$4s7s\ ^3S_1$	$4s4p\ ^3P_2$	9.417E–02			7.8E–02	20.73
$4s5d\ ^3D_1$	$4s4p\ ^3P_0$	1.927E–01			2.12E–01	–9.12
$4s5d\ ^3D_1$	$4s4p\ ^3P_1$	1.452E–01			1.53E–01	–5.10
$4s5d\ ^3D_1$	$4s4p\ ^3P_2$	9.773E–03			9.5E–03	2.87
$4s5d\ ^3D_2$	$4s4p\ ^3P_1$	2.593E–01			2.97E–01	–12.69
$4s5d\ ^3D_2$	$4s4p\ ^3P_2$	8.734E–02			9.4E–02	–7.08
$4s5d\ ^3D_3$	$4s4p\ ^3P_2$	3.451E–01			3.55E–01	–2.78
$4s4f\ ^3F_2$	$3d4s\ ^3D_1$	1.348E–01			1.76E–01	–23.43
$4s4f\ ^3F_2$	$3d4s\ ^3D_2$	2.508E–02			3.5E–02	–28.33
$4s4f\ ^3F_2$	$3d4s\ ^3D_3$	7.142E–04			9.8E–04	–27.13
$4s4f\ ^3F_3$	$3d4s\ ^3D_2$	1.429E–01			2.09E–01	–31.63
$4s4f\ ^3F_3$	$3d4s\ ^3D_3$	1.791E–02			2.5E–02	–28.38
$4s4f\ ^3F_4$	$3d4s\ ^3D_3$	1.617E–01			2.29E–01	–29.40
$4s5p\ ^3P_2$	$3d4s\ ^3D_1$	6.273E–04			2.3E–03	–72.72
$4s5p\ ^3P_2$	$3d4s\ ^3D_2$	1.558E–02			3.3E–02	–52.79
$4s5p\ ^3P_2$	$3d4s\ ^3D_3$	8.710E–02			1.9E–01	–54.16
$4s5p\ ^3P_1$	$3d4s\ ^3D_1$	2.532E–02			5.6E–02	–54.79
$4s5p\ ^3P_1$	$3d4s\ ^3D_2$	7.859E–02			1.7E–01	–53.77
$3d4p\ ^3F_4$	$3d4s\ ^3D_3$	3.850E–01			5.3E–01	–27.36
$4s5p\ ^3P_0$	$3d4s\ ^3D_1$	1.045E–01			2.2E–01	–52.48
$3d4p\ ^3F_3$	$3d4s\ ^3D_2$	3.367E–01			4.7E–01	–28.36
$3d4p\ ^3F_3$	$3d4s\ ^3D_3$	4.447E–02			5.9E–02	–24.62
$3d4p\ ^3F_2$	$3d4s\ ^3D_1$	2.250E–01			4.4E–01	–48.86
$3d4p\ ^3F_2$	$3d4s\ ^3D_2$	4.840E–02			8.1E–02	–40.25
$3d4p\ ^3F_2$	$3d4s\ ^3D_3$	7.591E–04			2.4E–03	–68.37
$4s5s\ ^3S_1$	$4s4p\ ^3P_0$	9.246E–02			9.6E–02	–3.69
$4s5s\ ^3S_1$	$4s4p\ ^3P_1$	2.759E–01			2.87E–01	–3.88
$4s5s\ ^3S_1$	$4s4p\ ^3P_2$	4.549E–01			4.77E–01	–4.63