Calculations of the neutron skin and its effect in atomic parity violation

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We perform calculations for the neutron skin of nuclei and its contribution to atomic parity nonconservation (PNC) in many isotopes of Cs, Ba, Sm, Dy, Yb, Tl, Pb, Bi, Fr, and Ra. Three problems are addressed: (i) neutron-skin-induced errors to single-isotope PNC, (ii) the possibility of measuring neutron skin using atomic PNC, and (iii) neutron-skin-induced errors for ratios of PNC effects in different isotopes. In the latter case the correlations in the neutron skin values for different isotopes lead to cancellations of the errors; this makes the isotopic ratio method a competitive tool in a search for new physics beyond the standard model.

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I. INTRODUCTION

Atomic parity nonconservation (PNC) provides powerful constraints on extensions to the standard model (SM) of elementary particles in the low-energy electroweak sector. In such measurements one determines a parity-violating signal E_{PNC}, related to the quantity of interest, the weak charge, Q_W, as $E_{\mathrm{PNC}} = k_{\mathrm{PNC}} Q_W$. The coefficient k_{PNC} comes from atomic calculations. With consideration of the challenges faced by such calculations, an alternative approach was proposed in Ref. [1]. The idea was to form a ratio $\mathcal{R}$ of the PNC amplitudes for two isotopes of the same element. Since the factor k_{PNC} remains the same, it cancels out in the ratio. However, in Ref. [2] a conceptual limitation to this approach was pointed out—an enhanced sensitivity of possible constraints on "new physics" to uncertainties in the *neutron* distributions. This problem is usually referred to as the problem of the neutron "skin." This problem has persisted for almost for two decades. Here we show that the neutron skins in different isotopes are correlated; this leads to a substantial cancellation in the neutron-skin-induced uncertainties in the PNC ratios. The use of modern experimental data and nuclear calculations makes the isotopic ratio method a competitive tool in the search for new physics beyond the SM.

The neutron skin ΔR_{np} is defined as a difference between the root-mean-square radii R_n and R_p of neutron and proton distributions. Even in interpreting the most accurate to date single-isotope measurement in Cs [3], this was a point of concern, as the induced uncertainty was comparable to the experimental error bar for the PNC amplitude [4,5]. The question was addressed in Ref. [6], where empirical antiprotonic atom data fit for the neutron skin was used [7] and the associated uncertainty in the "skin" contribution to E_{PNC} was substantially reduced. Meanwhile this question has yet to be settled for ongoing PNC experiments with unstable analogs of Cs: Fr [8] and Ra$^+$ [9]. In this article we use results of recent advances in our theoretical and experimental understanding of neutron skins to address these important questions.

II. NUCLEAR-STRUCTURE CALCULATIONS

There have been several recent advances in theoretical models and experimental investigations on neutron skins that warrant a new look at its impact on atomic parity violation. Historically, hadron-scattering experiments have provided the first indication of the neutron skin. For example, a value of $R_n = 0.17$ fm for ^{208}Pb was found to be consistent with proton scattering angular data [10,11]. But the uncertainty from many-body strong interaction effects is difficult to quantify [12]. Recently, antiprotonic atom data have been analyzed with a variety of nuclear energy-density functionals [13] including those for the Skyrme and relativistic Hartree formulations. This analysis led to the first set of Skyrme models (Skxs15, Skxs20, and Skxs25) [13] in which all parameters were adjusted to a global set of nuclear data that includes a realistic range of values for the neutron skin of ^{208}Pb, $R_n = 0.20(5)$. This new energy-density-functional model allows one to relate the value of the neutron skin for ^{208}Pb (and its uncertainty) to predictions over the entire range of nuclei required for comparison to other experiments and required for our atomic PNC calculations. The neutron skin can also be constrained by the properties of the pygmy dipole resonance in neutron-rich nuclei [14]. Data for ^{132}Sn suggest a value of $\Delta R_{np} = 0.24(4)$ [15]. This is consistent with the value of 0.27(5) fm obtained for ^{132}Sn from the Skxs20(5) interactions. The uncertainty in the value of the neutron skin will be reduced over the coming years, for example, from the PREX experiment at JLAB [16] by using a PNC asymmetry in elastic scattering of electrons from ^{208}Pb to measure R_n to a 1% ($\pm$0.05 fm) accuracy.

The neutron skin in a nucleus with a neutron excess depends on properties of the symmetry term of the potential that is the source of the uncertainty. The neutron skin in a nucleus with $N = Z$ (such as ^{100}Sn) is essentially well fixed (at a small negative value) by the Coulomb potential. Thus, when the symmetry potential part of the energy-density functional is fixed by the neutron skin for some neutron-rich heavy nucleus (such as ^{208}Pb), this functional can be used to obtain values for the neutron skin over the entire range of nuclei required for our

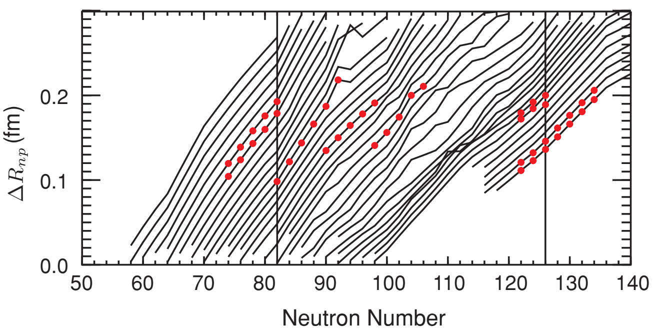

FIG. 1. (Color online) The neutron skin ΔR_{np} for nuclei above ^{100}Sn (all Z, even N values). Different isotopes for a given element are connected by lines. The filled circles are those for the nuclei of interest for atomic parity violation.

atomic PNC calculations. The Skxs20(5) functionals allow us to calculate the neutron skin in all nuclei with a well-defined value and error for each nucleus that can be compared with data.

The neutron skins for nuclei above ^{100}Sn obtained with Skxs20 are shown in Fig. 1. These are obtained in a spherical basis with nucleons allowed to occupy the lowest energy orbitals (with a calculation that does not have deformation or pairing). The irregularities between the neutron magic numbers 82 and 126 are due to the filling of the proton $h_{11/2}$ orbital and the neutron $i_{13/2}$ orbital. However, since the orbitals in the major shells $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $s_{1/2}$, and $h_{11/2}$ for protons and $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $i_{13/2}$ for neutrons are closely spaced, deformation and pairing will average out these irregularities. Deformation has a relatively large effect on the matter rms radii, but it has recently been shown that deformation itself does not have a strong effect on the neutron skin [17]. Thus, one can take the proton rms radii from the precise measurements together with a calculated value for the neutron skin to obtain the neutron radius required for atomic PNC. Thus, we discuss our atomic results in a model where the proton rms radius and neutron skin value appear separately. These irregularities depend on deformation and pairing, which can be implemented with much more computationally intensive calculations, but our spherical calculations containing the information on neutron skins are the essential new aspect of this paper.

III. ATOMIC PNC AND NEUTRON SKIN

The PNC observables depend on matrix elements of weak interaction [18],

$$\langle j|H_W|i\rangle = \frac{G_F}{2\sqrt{2}} C_{ji} R_p^{2\gamma-2} \bar{Q}_W, \quad (1)$$

where factor C_{ji} depends on atomic wave functions, $\gamma = \sqrt{1-(\alpha Z)^2}$, and $\bar{Q}_W$ includes the dependence on nuclear distributions,

$$\bar{Q}_W = -N q_n + Z q_p (1 - 4\sin^2\theta_W) + \Delta Q_{\text{new}}. \quad (2)$$

The term ΔQ_{new} characterizes new physics and θ_W is the Weinberg angle. The quantities q_n and q_p depend on the neutron and proton distributions convoluted with atomic wave functions: $q_n = 1 + f_n(\frac{R_n}{R_p})$. In the "sharp-edge" model of the nuclear density distribution,

$$f_n\left(\frac{R_n}{R_p}\right) \approx -\frac{3}{70}(\alpha Z)^2 \left[1 + 5\left(\frac{R_n}{R_p}\right)^2\right]. \quad (3)$$

The accuracy of theis formula is sufficient for the present goals [19].

IV. SINGLE-ISOTOPE MEASUREMENTS

The relative correction to the PNC amplitude from the neutron skin reads [6]

$$\frac{\delta E_{\text{PNC}}^{\text{n.s.}}}{E_{\text{PNC}}} = -\frac{3}{7}(\alpha Z)^2 \frac{\Delta R_{np}}{R_p}. \quad (4)$$

The computed corrections for all the isotopes are listed in Table I. In particular, for ^{133}Cs the relative correction is $-0.0023(5)$, which is consistent with the value of $-0.0019(8)$ from Ref. [6], which was based on the semi-empirical fit of antiprotonic atom data [7] $\Delta R_{np} = 0.13(4)$ fm]. As we progress to heavier elements, the correction grows as Z^2, reaching 0.6% for Fr and Ra$^+$. As an example, for ^{213}Fr, the correction reads $-0.0063(16)$. The error bar implies that, at the present level of knowing the neutron skin, it contributes to the uncertainty in the extraction of new physics from the Fr experiment at the 0.1%–0.2% level.

V. DETECTING NEUTRON SKIN IN ATOMIC PNC

The question of determining neutron skin is of interest in its own right, for example, for the equation of state for neutron stars. It is worth mentioning the proposed PREX experiment at JLAB [16] in which a PNC asymmetry in elastic scattering of electrons from ^{208}Pb is used to measure R_n to a 1% ($\pm$0.05 fm) accuracy. There have also been renewed attempts to obtain R_n from hadronic scattering data [10,11].

Considering this interest, we would like to see whether the neutron skin can be extracted from atomic PNC measurements. From the preceding discussion, it is clear that for the single-isotope PNC the uncertainty of experiments and atomic calculations should be smaller than 0.2% (Cs, Ba$^+$) and 0.6% (Fr, Ra$^+$). This seems to be a realistic goal [20,21].

This problem can also be addressed in the isotopic chain experiments. Suppose the PNC amplitudes E_{PNC} and E'_{PNC} are measured for two isotopes of the same atom or ion with neutron numbers N and $N' = N + \Delta N$ and the ratio is formed:

$$\mathcal{R} = \frac{E_{\text{PNC}}}{E'_{\text{PNC}}} = \frac{\bar{Q}_W}{\bar{Q}'_W}\left(\frac{R_p}{R'_p}\right)^{2\gamma-2}. \quad (5)$$

Here all quantities with primes are for the isotope with N' neutrons. Focusing on the contribution of the neutron skin, we have

$$\mathcal{R} \approx \frac{N}{N'}\left(\frac{R_p}{R'_p}\right)^{2\gamma-2} \times \left\{1 + \left[f_n\left(\frac{R_n}{R_p}\right) - f_n\left(\frac{R'_n}{R'_p}\right)\right]\right\}.$$

TABLE I. Computed neutron skins ΔR_{np} and proton distribution rms radii. The relative contributions of the neutron skin to atomic PNC amplitudes are listed in the last column.

A	ΔR_{np} (fm)	R_p (fm)	$\delta E_{\rm PNC}^{\rm n.s}/E_{\rm PNC}$
	Cs ($Z = 55$)		
129	0.120(32)	4.705	−0.0018(5)
131	0.139(34)	4.714	−0.0020(5)
133	0.158(37)	4.723	−0.0023(5)
135	0.176(40)	4.732	−0.0026(6)
137	0.193(42)	4.742	−0.0028(6)
	Ba ($Z = 56$)		
130	0.104(29)	4.723	−0.0016(4)
132	0.124(32)	4.731	−0.0019(5)
134	0.143(34)	4.740	−0.0022(5)
136	0.161(37)	4.749	−0.0024(6)
138	0.179(40)	4.759	−0.0027(6)
	Sm ($Z = 62$)		
144	0.098(27)	4.894	−0.0018(5)
146	0.122(32)	4.893	−0.0022(6)
148	0.144(35)	4.903	−0.0026(6)
150	0.166(39)	4.913	−0.0030(7)
152	0.187(43)	4.924	−0.0033(8)
154	0.219(48)	4.934	−0.0039(9)
	Dy ($Z = 66$)		
156	0.135(35)	4.997	−0.0027(7)
158	0.150(37)	5.018	−0.0030(7)
160	0.164(38)	5.039	−0.0032(8)
162	0.178(40)	5.061	−0.0035(8)
164	0.191(42)	5.082	−0.0037(8)
	Yb ($Z = 70$)		
168	0.141(35)	5.143	−0.0031(8)
170	0.153(38)	5.163	−0.0033(8)
172	0.174(40)	5.171	−0.0038(9)
174	0.202(51)	5.173	−0.0044(11)
176	0.215(67)	5.193	−0.0046(14)
	Tl ($Z = 81$)		
203	0.179(45)	5.422	−0.0049(12)
205	0.192(48)	5.434	−0.0053(13)
	Pb ($Z = 82$)		
204	0.172(44)	5.430	−0.0049(12)
206	0.184(46)	5.442	−0.0052(13)
208	0.200(50)	5.450	−0.0056(14)
	Bi ($Z = 83$)		
209	0.189(48)	5.468	−0.0054(14)
	Fr ($Z = 87$)		
209	0.121(36)	5.518	−0.0038(11)
211	0.132(38)	5.529	−0.0041(12)
213	0.146(42)	5.536	−0.0046(13)
215	0.161(44)	5.546	−0.0050(14)
217	0.176(47)	5.555	−0.0055(15)
219	0.191(50)	5.565	−0.0059(16)
221	0.206(53)	5.574	−0.0064(16)
	Ra ($Z = 88$)		
210	0.111(34)	5.535	−0.0035(11)
212	0.123(37)	5.546	−0.0039(12)
214	0.136(40)	5.553	−0.0043(13)
216	0.151(43)	5.563	−0.0048(14)
218	0.166(46)	5.572	−0.0053(15)
220	0.181(49)	5.581	−0.0057(16)
222	0.195(52)	5.591	−0.0062(16)

Neglecting the neutron skin ($R_n \to R_p$) gives $\mathcal{R} \to \mathcal{R}_0 \equiv N/N'(R_p/R'_p)^{2\gamma-2}$. Any deviation of $\mathcal{R}$ from $\mathcal{R}_0$ is a signature of the neutron skin. The figure of merit is

$$\Delta\mathcal{R}_{\rm n.s.} = (\mathcal{R} - \mathcal{R}_0)/\mathcal{R}_0 = f_n\left(\frac{R_n}{R_p}\right) - f_n\left(\frac{R'_n}{R'_p}\right), \quad (6)$$

where f_n is given by Eq. (3). In terms of the neutron skin then $\Delta\mathcal{R}_{\rm n.s.} \approx \frac{3}{7}(\alpha Z)^2 \frac{1}{R_p}[\Delta R'_{np} - \Delta R_{np}]$.

There are two observations that can be made: (i) The isotopic ratios are sensitive to the differential change in the skin thickness (i.e., the neutron skin effects tend to cancel) and (ii) the largest effect is attained for a pair of isotopes where the skin thicknesses differ the most. This condition is reached for a pair comprising the lightest (neutron-depleted) and the heaviest (neutron-rich) isotopes of the chain.

For Cs, Ba, and Dy $\Delta\mathcal{R}_{\rm n.s} \approx 0.001$, for Yb and Sm $\Delta\mathcal{R}_{\rm n.s} \approx 0.002$, and for Fr and Ra $\Delta\mathcal{R}_{\rm n.s} \approx 0.003$. Fr and Ra are the extreme cases, as the skin thicknesses for the lightest and the heaviest isotopes differ by a factor of 2.

VI. ISOTOPIC RATIOS: NEUTRON SKIN VERSUS NEW PHYSICS

Suppose we form the ratio of measured PNC amplitudes for two isotopes of the same atom or ion. Can we constrain new physics beyond the SM by analyzing the ratios? The previous studies have answered this question negatively, as the uncertainties in the neutron skin masked the new physics contributions. In the following, in light of our nuclear calculations, we revisit this question. The discussion follows previous analysis [22]. There are two new points: (i) We use more finely tuned calculations of the skin (Table I in lieu of the empirical fit of antiprotonic atom data [7]) and (ii) we take into account that the errors in ΔR_{np} for two isotopes are correlated. Both these factors allow us to argue that, by contrast to the previous studies, the isotopic ratios can provide competitive constraints on the new physics.

The term $\Delta Q_{\rm new}$ in Eq. (2) characterizes new physics at the tree level. Following Ref. [23], we represent it as a combination of new-physics couplings to protons and neutrons, $\Delta Q_{\rm new} \equiv Zh_p + Nh_n$. Various elementary-particle scenarios for these interactions were reviewed in Ref. [23]. Then $\bar{Q}_W = Nh_0 + Zh_p + Nh_n$, where h_0 comes from the SM (which also includes the nuclear corrections). Unlike in the single-isotope measurements (which are sensitive mainly to h_n), in the isotopic ratio method, the sensitivity to new

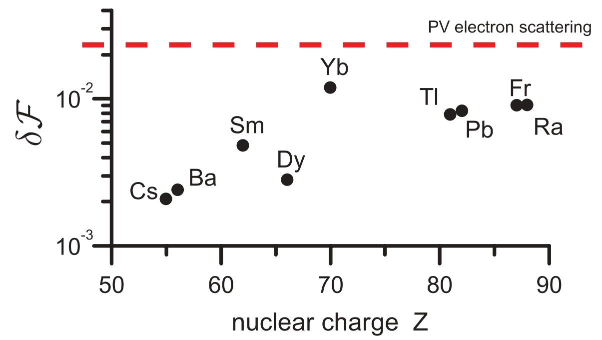

FIG. 2. (Color online) The neutron-skin-induced uncertainties, $\delta\mathcal{F}$, for isotopic chains compared with the constraints from parity-violating electron scattering, $\mathcal{F}$.

TABLE II. Contribution of nuclear-structure uncertainty to a constraint on "new physics" $\delta\mathcal{F}$ obtained from a smoothed variation in the neutron skin with neutron number.

Atom	Mass numbers A		$\delta\mathcal{F}\times 10^3$
Cs ($Z=55$)	129	137	2.1
Ba ($Z=56$)	130	138	2.3
Sm ($Z=62$)	144	154	4.2
Dy ($Z=66$)	156	164	2.7
Yb ($Z=70$)	168	176	10.2
Tl ($Z=81$)	203	205	7.2
Pb ($Z=82$)	204	208	7.7
Fr ($Z=87$)	209	221	8.8
Ra ($Z=88$)	210	222	8.9

physics comes predominantly from h_p [23]. The sensitivity reads [22]

$$\mathcal{F}=\frac{h_p}{h_0}=\left(\frac{\mathcal{R}}{\mathcal{R}_0}-1\right)\frac{NN'}{Z\Delta N}. \quad (7)$$

In the absence of new couplings $\mathcal{F}=0$. The smaller the value of $\mathcal{F}$, the tighter the constraints on h_p are. For a given chain, it is beneficial to work with the largest possible neutron spread ΔN (i.e., forming the pairs from the lightest and the heaviest elements of the chain).

The constraints on h_p, Eq. (7), are affected by (i) the experimental errors, $\delta\mathcal{R}_{\rm exp}$, and (ii) uncertainties in $\mathcal{R}_0$ that are induced by insufficient knowledge of nuclear distributions. Explicitly,

$$\delta\mathcal{F}=\frac{NN'}{Z\Delta N}\left\{\frac{\delta\mathcal{R}_{\rm exp}}{\mathcal{R}_0}+\delta[f_n-f_n']\right\}, \quad (8)$$

where $\delta[\cdots]$ stands for variation. Explicitly,

$$\delta[f_n-f_n']\approx\frac{3}{14}(\alpha Z)^2\,\delta\left[\frac{R_n'^2}{R_p'^2}-\frac{R_n^2}{R_p^2}\right]. \quad (9)$$

An important point is that the neutron skin errors in Eq. (9) for two isotopes are correlated. From numerical experimentation, we find that for a given isotopic chain a variation of nuclear interaction parameters induces similar change in the ratio R_n/R_p for all the isotopes. Indeed, in Table I the boundaries of error bars in the values of neutron skin correspond to the same values of nuclear parameters. We see that the errors tend to cancel each other. This is to be contrasted with the previous analysis of Ref. [22], in which the errors from individual isotopes were treated independently (i.e., errors were added in quadrature). We find that our new "correlated" treatment reduces the error bars by a factor of 4–10.

We calculated $\delta\mathcal{F}$ from a smoothed variation in the neutron skin with neutron number over pairs of adjacent isotopes ($\Delta N=2$). The results are compiled in Table II and also in Fig. 2. These errors are compared to the new-physics couplings to protons, h_p, which are directly probed by the parity-violating electron scattering (PVES). The PVES experiments were recently analyzed in Ref. [24]. The resulting weak charge of the proton is $Q_W^p=0.058\pm0.023$ [25] and is in agreement with the SM value [26]. The error bar determines the upper bound on $\mathcal{F}$ shown in Fig. 2. It is clear that all isotopic-chain determinations are competitive to bounds derived from PVES. For example, measurements with isotopes of Cs, Ba, and Dy would be an order of magnitude more sensitive to the new physics. The situation may change as a future Q-weak PVES experiment [27] could improve bounds on Q_W^p by an order of magnitude. If that experiment discovers some new physics, atomic PNC with Cs, Ba, and Dy would provide independent tests of the Q-weak data. Finally, we emphasize that the errors $\delta\mathcal{F}$ in Table II are based on spherical Skyrme calculations. It is important to study these errors further with additional calculations explicitly including deformation and pairing.

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