"Doubly Magic" Conditions in Magic-Wavelength Trapping of Ultracold Alkali-Metal Atoms

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In experiments with trapped atoms, atomic energy levels are shifted by the trapping optical and magnetic fields. Regardless of this strong perturbation, precision spectroscopy may be still carried out using specially crafted, ''magic'' trapping fields. Finding these conditions for particularly valuable microwave transitions in alkali-metal atoms has so far remained an open challenge. Here I demonstrate that the microwave transitions in alkali-metal atoms may be indeed made impervious to both trapping laser intensity and fluctuations of magnetic fields. I consider driving multiphoton transitions between the clock levels and show that these ''doubly magic'' conditions are realized at special values of trapping laser wavelengths and fixed values of relatively weak magnetic fields. This finding has implications for precision measurements and quantum information processing with qubits stored in hyperfine manifolds.

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Compared to spectroscopic beam experiments, trapping cold atoms and molecules removes Doppler shifts and increases interrogation time, thereby dramatically enhancing spectral resolution. This improvement comes at a price: trapping optical fields strongly perturb atomic energy levels—transition frequencies are shifted away from their unperturbed values. In addition, the underlying Stark shift is proportional to the local intensity of the trapping lasers; the shift is nonuniform across the atomic ensemble and it is also sensitive to laser intensity fluctuations. So trapping seems to be both advantageous and detrimental for precision measurements. This dilemma is elegantly solved using so-called magic traps [1]. At the magic trapping conditions two levels of interest are shifted by exactly same amount by the trapping fields; therefore the differential effect of trapping fields simply vanishes for that transition.

The idea of such magic trapping has been crucial for establishing a new class of atomic clocks [2], the optical lattice clocks. Here atoms are trapped in optical lattices formed by counterpropagating laser beams; the lasers operate at the magic wavelength. In these clocks one employs optical transitions in divalent atoms, such as Sr and Yb. Finding similar magic conditions for ubiquitous alkali-metal atoms employed in a majority of cold-atom experiments remains an open challenge. Especially valuable are the microwave transitions in the ground-state hyperfine manifold. Identifying magic conditions here, for example, would enable developing micromagic clocks [3]: lattice clocks operating in the microwave region of the spectrum. In addition, the hyperfine manifolds are used to store quantum information in a large fraction of quantum computing proposals with ultracold alkali-metal atoms. Finding magic conditions would enable a decoherence-free trapping for this important realization of qubits.

The clock transitions in divalent atoms are between nonmagnetic states; this removes sensitivity to magnetic fields. For alkali-metal atoms, however, an additional piece of the puzzle is that the clock or qubit states are sensitive to both optical and magnetic fields. One needs to eliminate the sensitivity of transition frequency ν to both perturbations simultaneously. This problem is solved here. We will require that the clock-qubit transition is insensitive to both Stark- and Zeeman-induced perturbations (we will use the doubly magic qualifier for such trapping conditions).

First steps in identifying magic conditions for hyperfine transitions in alkali-metal atoms have been made in Refs. [4–6]. These works focused on eliminating sensitivity to laser intensity I_L by tuning the laser frequency to its magic value ω_m, $\delta\nu(\omega_m) = 0$, regardless of the value of I_L. The proposals [5,6] have neglected the effect of magnetic fields and focused on B field sensitive $M_F \neq 0$ states. So while the trapping would be Stark-insensitive, the states would decohere due to coupling to stray B fields. A partial

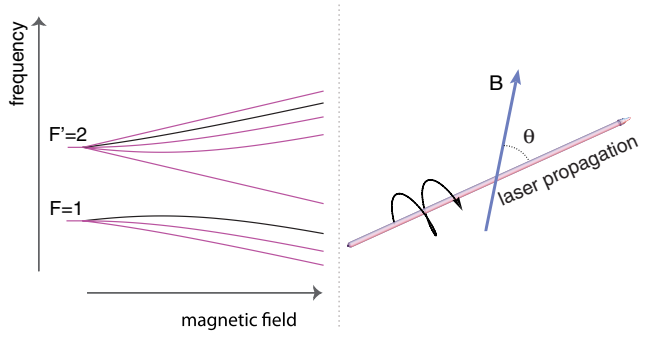

FIG. 1 (color online). Left panel: Zeeman effect for the hyperfine manifold in the ground state of $I = 3/2$ isotopes of alkali-metal atoms. Two clock or qubit levels $|F' = 2, M'_F = +1\rangle$ and $|F = 1, M_F = -1\rangle$ are shown in black. The right panel illustrates the geometry of laser-atom interaction: the degree of circular polarization, angle θ, and laser wavelength may be varied.

TABLE I. Values of magic B fields and magic wavelengths.

Transition	B_m, Gauss	λ_m
^{87}Rb, $I = 3/2$, $\nu_{\text{clock}} = 6.83$ GHz		
$\|2, 1\rangle \rightarrow \|1, -1\rangle$	3.25	806 nm[a]
^{85}Rb, $I = 5/2$, $\nu_{\text{clock}} = 3.04$ GHz		
$\|3, 1\rangle \rightarrow \|2, -1\rangle$	0.359	
$\|3, 2\rangle \rightarrow \|2, -2\rangle$	1.15	479–658;797–878
^{133}Cs, $I = 7/2$, $\nu_{\text{clock}} = 9.19$ GHz		
$\|4, 1\rangle \rightarrow \|3, -1\rangle$	1.41	
$\|4, 2\rangle \rightarrow \|3, -2\rangle$	3.51	906–1067; 560-677
$\|4, 3\rangle \rightarrow \|3, -3\rangle$	9.04	898–1591;863–880; 512–796

[a]Nearly doubly magic.

solution to this problem was discussed in Refs. [7,8]: moving to the $M_F = 0$ states eliminates sensitivity to Zeeman shifts to the leading order. Yet one needs to apply a bias magnetic field of a specific value making the conditions magic for a given trapping laser wavelength. As a result, the transitions remain Zeeman-sensitive through the second-order effects; numerical estimates show that, unfortunately, the residual B field sensitivity would preclude designing a competitive clock.

Clearly, the sensitivity to B fields has to be addressed. We require that at the magic B field $d\nu/dB(B_m) = 0$. Such conditions occur, for example, for a two-photon $|F' = 2, M'_F = +1\rangle \rightarrow |F = 1, M_F = -1\rangle$ transition in ^{87}Rb at the field of about 3 Gauss. The relevant Breit-Rabi diagram is shown in Fig. 1. The two clock or qubit levels are highlighted: the existence of the magic B field may be inferred visually. Experimentally, such magic conditions have been proven instrumental for performing collisional studies in magnetically trapped Bose-Einstein condensates of ^{87}Rb [9] and studying the decoherence of a cold cloud of Rb near a microchip [10]. Similar ideas are essential to realizing atomic clocks on a microchip [11,12]. My present work adds the optical fields to the mix: it turns out that the unified Stark-Zeeman description is nontrivial due to an interference of the two effects.

Formalism.—We are interested in the transition between two hyperfine states $|F' = I + 1/2, M'_F\rangle$ and $|F = I - 1/2, M_F\rangle$ attached to the ground electronic $nS_{1/2}$ state of an alkali-metal atom (I is the nuclear spin). Here and below we denote the upper state as $|F'\rangle$ and the lower state as $|F\rangle$ (see Fig. 1).

We focus on the $M'_F = -M_F$ transitions. For these transitions, the electronic g factors of the two states are the same (see Fig. 1). Then the bulk of the Zeeman shift of the transition frequency goes away and the linear Zeeman effect is determined only by the nuclear g factor $g_I = 1/I\mu_{\text{nuc}}/\mu_N$, where μ_N is the nuclear magneton. The residual linear shift is compensated by the second-order Zeeman correction, quadratic in the B field. This leads to the magic value of the B field

$$B_m \approx \frac{g_I \mu_N M_{F'}}{2|\langle F, M_{F'}|\mu_z^e|F', M_{F'}\rangle|^2} h\nu_{\text{clock}}, \tag{1}$$

where μ^e is the operator of the magnetic moment of an electron. The second-order estimate, Eq. (1), is a good approximation: for ^{87}Rb it gives 3.25 G, while the "all-order" Breit-Rabi analysis yields $B_m = 3.228\,917(3)$ G (Ref. [9]). Values of B_m for Rb and Cs isotopes are tabulated in Table I. Notice that the fields for all tabulated transitions are relatively weak and can be well stabilized using existing technologies [11].

Fixing the magnetic field at its magic value accomplishes the Zeeman-insensitivity of the clock-qubit transitions. Now we would like to additionally remove the Stark sensitivity to intensity of trapping laser fields. We consider the following setup shown in Fig. 1. An atom is illuminated by a laser light, with a certain degree of circular polarization A. At the same time, a bias magnetic field is applied at an angle θ to the direction of laser propagation. The B field is fixed at its magic value. This is a basic building block for optical trapping. For example, an optical lattice (standing wave) may be formed by two counterpropagating lasers of the same wavelength.

In a laser field, both clock or qubit levels are shifted due to the dynamic (i.e., laser-frequency-dependent) Stark effect [13]. I derived the following formula for the differential shift of the clock or qubit frequency [14],

$$\delta\nu_{\text{clock}}(\omega) = -\frac{1}{h}\Big\{(\beta^s_{F'} - \beta^s_F) + A\cos\theta M_{F'}\Big[\Big(\frac{1}{2F'}\beta^a_{F'} + \frac{1}{2F}\beta^a_F\Big) + g_I\frac{\mu_N}{\mu_B}\alpha^a_{nS_{1/2}}\Big]\Big\}\Big(\frac{E_L}{2}\Big)^2. \tag{2}$$

Here E_L is the amplitude of the laser E field, $I_L \propto E_L^2$. The quantities β^s, β^a, and α^a are the scalar and vector (axial) polarizabilities (see below). An important fact is that all these polarizabilities depend on the laser frequency, ω. By tuning the laser frequency we require that the combination in curly bracket becomes zero. At that magic point, the differential shift vanishes independently of the laser intensity: $\delta\nu_{\text{clock}}(\omega_m) = 0$.

What is the difference between the polarizabilities α and β? We are considering the Stark shift of hyperfine levels attached to the same electronic state. To the leading order, the shift is determined by the properties of the underlying electronic state (polarizability α). However, because the electronic state for both hyperfine levels is the same, the levels are shifted at the same rate [15] and we need to distinguish between the two hyperfine levels. An apparent difference between the two clock or qubit levels is caused by the hyperfine interaction, and the rigorous analysis involves so-called hyperfine-interaction-mediated polarizabilities, β. Lengthy third-order (two dipole couplings to

the laser field and one hyperfine interaction) expressions for these polarizabilities may be found in Ref. [4].

Continuing with our discussion of the frequency shift (2), I would like to stress an unconventional origin of the last contribution. Its full form for an arbitrary B field is $A\cos\theta M_{F'} g_I \frac{\mu_N}{\mu_B} \alpha^a_{nS_{1/2}}(B/B_m)$. The term arises due to an interference between Stark and Zeeman interactions. Qualitatively, the vector contribution to the Stark shift has the very same rotational properties as the Zeeman coupling (both are vector operators). These operators, in particular, couple the two hyperfine manifolds. Consider the shift of the $|F', M_F\rangle$ level. The Zeeman operator couples it to the $|F, M_F\rangle$ intermediate state, and then the vector Stark shift operator brings it back to the $|F', M_F\rangle$ level, thereby resulting in the energy shift. This cross term is of the same order of magnitude as the other two terms in Eq. (2) and has to be included in the consideration.

Now we can find magic wavelengths by numerically evaluating atomic polarizabilities entering Eq. (2). To this end I used a blend of relativistic many-body techniques of atomic structure, as described in [16]. To improve upon the accuracy, high-precision experimental data were used where available. To ensure the quality of the calculations, a comparison with the experimental literature data on static Stark shifts of the hyperfine transitions was made. It is one of the problems where a consistent treatment is important and less sophisticated estimates may fail even qualitatively (see [8] for a discussion).

Results.—I will present the results of the calculations in the following form. Since the magic condition corresponds to $\delta\nu_{\rm clock}(\omega_m) = 0$, we may recast Eq. (2) into

$$M_{F'} A\cos\theta = -\frac{\beta^s_{F'} - \beta^s_F}{(\frac{1}{2F'}\beta^a_{F'} + \frac{1}{2F}\beta^a_F) + g_I \frac{\mu_N}{\mu_B}\alpha^a_{nS_{1/2}}}. \quad (3)$$

The right-hand side of this equation depends on the laser frequency, while the left-hand side does not. Moreover, $|A\cos\theta| \leq 1$; therefore the magic conditions would exist only if for a given ω the right-hand side is within the range $-|M'_F|$ and $|M'_F|$.

Doubly magic trapping of ^{133}Cs is analyzed in Fig. 2. This atom is metrologically important. For the past four decades the SI unit of time, the second, has been defined as a duration of a certain number of periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the ^{133}Cs atom. Cs clocks serve as primary frequency standards worldwide and there is a substantial investment in the infrastructure supporting these clocks.

From Fig. 2 we find that the doubly magic trapping of Cs atoms is indeed possible for two transitions: $|4, 2\rangle \rightarrow |3, -2\rangle$ and $|4, 3\rangle \rightarrow |3, -3\rangle$. The only complication is that driving the former transition requires 4 photons, while the latter transition requires 6 photons. This may be potentially accomplished either with multistep rf or MW or stimulated Raman drives [9,17]. In addition, at least for the two-photon transitions, an electromagnetically-induced-transparency-based interrogation scheme that partially removes sensitivity to the probing fields was proposed recently [18].

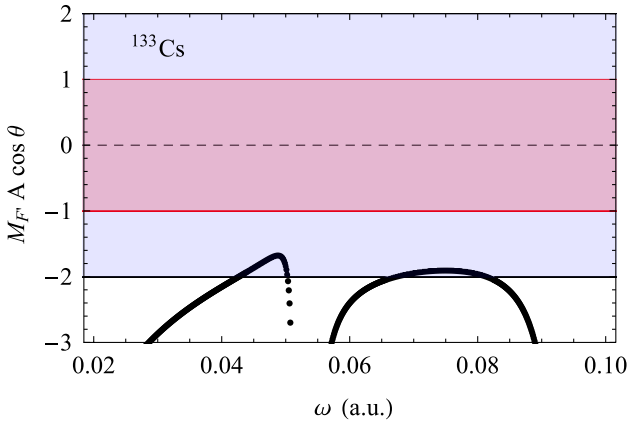

FIG. 2 (color online). Magic conditions for ^{133}Cs. A dependence of the product $M_{F'}A\cos\theta$ on trapping laser frequency (in atomic units) is plotted. The shaded regions are bound by $-|M_{F'}|$ and $+|M_{F'}|$ lines. Magic trapping for the $|F' = 4, M'_F\rangle \rightarrow |F = 3, -M'_F\rangle$ clock-qubit transition is only possible when the computed curve lies inside the corresponding shaded region.

The curve (3) exhibits a resonant behavior when the laser frequency passes through the fine-structure doublet of atomic transitions: $6s_{1/2}$-$6p_{1/2}$ and $6s_{1/2}$-$6p_{3/2}$ at $\omega =$ 0.050 932 a.u. and 0.053 456 a.u.. Not shown in the Fig. 2 are the values of the right-hand side of Eq. (3) between the resonances. In this region, the right-hand side values become positive and the curve crosses the $M_{F'}A\cos\theta = +3$ limit from above: the 6-photon transition may be made doubly magic when the laser is tuned to inside the fine-structure doublet.

^{87}Rb serves as the secondary frequency standard. An analysis of magic conditions for this atom is carried out in Fig. 3. There is a single, $|F' = 2, 1\rangle \rightarrow |F = 1, -1\rangle$, transition of interest here. This is a two-photon transition. Curiously, the $M_{F'}A\cos\theta$ curve nearly touches its limiting value at $\omega \approx 0.0565$ a.u. ($\lambda_m \approx 806$ nm) somewhat below the $5s_{1/2}$-$5p_{3/2}$ resonance. Here the right-hand side of Eq. (3) reaches values of ≈ -1.05; i.e., it is just 5% off the limiting value of -1. While not quite achieving the doubly magic status, this 806 nm wavelength gets us to nearly magic conditions. In practice, if one may afford small Zeeman decoherences, the bias B field may be detuned off its magic value. Then (see discussion above) the last term in the Stark shift, Eq. (2), is rescaled by the ratio (B/B_m) and the magic condition for the Stark shift can be reached. For example, I computed that $\lambda_m \approx$ 806 nm becomes "Stark magic" at a B field of 3.45 G, i.e., just 6% larger than its "Zeeman-magic" value. Ultimately, the choice of B should be a matter of optimizing tolerances to both Stark and Zeeman-induced decoher-

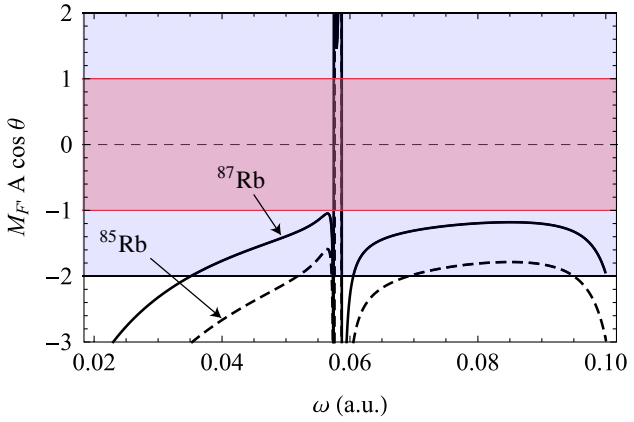

FIG. 3 (color online). Same as in Fig. 2 for ^{87}Rb transition $|F' = 2, 1\rangle \rightarrow |F = 1, -1\rangle$ and for ^{85}Rb transitions $|F' = 3, 1\rangle \rightarrow |F = 2, -1\rangle$ and $|F' = 3, 2\rangle \rightarrow |F = 2, -2\rangle$.

ence in a particular application. For example, in a recent Paris experiment [12] with a magnetically trapped ensemble of ^{87}Rb atoms, the bias B field has been varied by as much as 16% from its magic value; this still has led to well-resolved contrast on the $|F' = 2, 1\rangle \rightarrow |F = 1, -1\rangle$ clock transition.

Finally, in Fig. 3 we explore magic conditions for another isotope of Rb, ^{85}Rb. Compared to ^{87}Rb, the nuclear spin of this isotope is larger ($I = 5/2$); this results in larger values of F and richer magnetic substructure of the hyperfine levels. Two transitions of interest become available: $|F' = 3, 1\rangle \rightarrow |F = 2, -1\rangle$ and $|F' = 3, 2\rangle \rightarrow |F = 2, -2\rangle$. Moreover, various contributions to Eq. (2) scale differently with the nuclear spin and we need to carry out a separate calculation for each isotope. From Fig. 3, we find that doubly magic conditions can be attained for the $|F' = 3, 2\rangle \rightarrow |F = 2, -2\rangle$ transition in ^{85}Rb.

I also carried out calculations for other commonly used alkali-metal atoms. For ^{7}Li, ^{23}Na, and ^{39}K there are no doubly magic (or ''near-magic'') points; all these isotopes have $I = 3/2$. For various Fr isotopes, there is a multitude of doubly magic points for multiphoton transitions. For example, for the ^{210}Fr ($I = 6$) transition $M_F = -7/2 \rightarrow +7/2$ these conditions are attained in the range $\lambda_m =$ 846–1061 nm.

To conclude, by working at the doubly magic (Stark-Zeeman) conditions, one can greatly reduce sensitivity to spatial inhomogeneity due to trapping or bias fields and also reduce sensitivity to temporal fluctuations of the fields. It is anticipated that a variety of applications could take advantage of the magic (and nearly magic) conditions found in this Letter. For example, we anticipate that lifetimes of quantum memory [19] may be improved. Another opportunity is developing micromagic lattice clocks. While the detailed accuracy analysis for such clocks is beyond the scope of this Letter, the key finding here is that the multiphoton transitions in metrologically important ^{133}Cs can be made simultaneously insensitive both to intensities of trapping lasers and also to fluctuations of magnetic fields.

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