Enhanced cooling of hydrogen by a buffer gas of alkali-metal atoms

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We consider the possibility of enhanced cooling of hydrogen atoms by a buffer gas of alkali-metal atoms Na, K, Rb, and Cs. Ultracold elastic collision cross sections for the Na-H and Rb-H purely spin-polarized pairs are found to be 640 and 860 times larger than that for the H-H pair, respectively. From an analysis of the techniques of production of ultracold sodium and rubidium samples, it seems feasible that the critical conditions for Bose-Einstein condensation of hydrogen could be achieved already at the stage of optical cooling of the sodium or rubidium buffer gas.

Bose-Einstein condensation (BEC) of a dilute gas of hydrogen [1] is a delicate experimental task because of the smallness of elastic cross sections, which impedes evaporative cooling. Côté et al. [2] have investigated the possibility of an enhanced cooling of atomic hydrogen by mixing with ultracold lithium gas. They found that at ultracold temperatures the 7Li-H elastic cross section of purely spin-polarized pairs is 1400 times larger than for H-H collisions and therefore lithium could be efficient in accelerating the cooling of hydrogen. In the present work, we extend the treatment of Ref. [2] of the cooling of hydrogen in a buffer gas of lithium atoms to heavier Na, K, Rb, and Cs atoms. There is a particular interest in 23Na and 87Rb stemming from the availability of established methods to efficiently create ultracold samples and Bose-Einstein condensates [3,4]. By a direct calculation, we will demonstrate that the ultracold Na-H and Rb-H elastic cross section are a factor of 640 and 860 larger than that for a pair of hydrogen atoms, respectively. We find that with available techniques for creating ultracold sodium (rubidium) samples, the critical BEC conditions for hydrogen could be achieved through laser cooling of the alkali-metal component, without resorting to techniques of cryogenic and evaporative cooling.

In Ref. [5] we presented quantum-mechanical calculations of the potential-energy curves for the singlet and triplet states of LiH, NaH, KH, RbH, and CsH formed by the approach of ground-state alkali-metal atoms and hydrogen atoms. We determined precise values for the coefficients of the van der Waals interaction and estimated the contribution of the exchange interaction at large distances. Together with empirical data, these were used to assess and improve the accuracy of the ab initio potentials.

In a magnetic trap, atoms are confined in a “low-field seeking” spin-polarized state, and most ground-state collisions occur in the triplet molecular potential \( a^3\Sigma^+ \). In Fig. 1 we present the \( a^3\Sigma^+ \) molecular potentials between hydrogen and alkali-metal atoms, constructed in Ref. [5]. The potential wells at large internuclear distances are formed due to the interplay of repulsive exchange interaction and attractive van der Waals forces. Although at a given internuclear separation both interactions are stronger for a heavier dimer [5], the position of the minimum shifts towards larger distances as the mass of the alkali-metal atom increases. The potential well is deeper for lighter dimers. Each of these wells supports a single bound vibrational state. From the numerical solution of the eigenvalue problem, we determined the positions of the bound states and we present them in Table I.

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FIG. 1. \( a^3\Sigma^+ \) potentials for alkali hydrides [5].
Effective-range approximation, Eq. 3

\[ a_{\text{eff}} = \frac{\hbar}{\sqrt{2\mu E_b}}. \]  

Table I. Binding energy of the vibrational state supported by \( a^3\Sigma^+ \) potentials and the scattering lengths calculated in the effective-range approximation, Eq. (3). All values are in atomic units.

<table>
<thead>
<tr>
<th>Potential</th>
<th>( E_b ) (a.u.)</th>
<th>( a_{\text{eff}} ) (a.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23Na-H</td>
<td>2.964x10(^{-7})</td>
<td>31.0</td>
</tr>
<tr>
<td>39K-H</td>
<td>7.334x10(^{-7})</td>
<td>19.5</td>
</tr>
<tr>
<td>87Rb-H</td>
<td>2.443x10(^{-7})</td>
<td>33.6</td>
</tr>
<tr>
<td>133Cs-H</td>
<td>1.785x10(^{-7})</td>
<td>39.2</td>
</tr>
</tbody>
</table>

Rarity of the order of 1%, based on the agreement of theory and experiment for the singlet potentials [5].

Using the constructed potentials [5], we calculated radial continuum wave functions using the Numerov method, and obtained phase shifts \( \delta_l \), from which we determined scattering lengths, cross sections, and rate coefficients. The singlet and triplet scattering lengths \( a_S \) and \( a_T \) were calculated by taking the low-energy limit

\[ a = -\lim_{k \to 0} \frac{\tan \delta_0(k)}{k}. \]  

where \( \delta_0 \) is the s-wave phase shift, and \( k \) is the wave vector of relative motion. The values of \( a_S \) and \( a_T \) are listed in Table II. We tested the sensitivity of the scattering lengths by performing calculations with uniformly scaled potentials

\[ U(R) \to (1 + \lambda) U(R). \]  

The scattering lengths corresponding to \( \lambda = \pm 1\% \) are presented in Table II. The triplet scattering lengths are insensitive to such modifications, because the shallow \( a^3\Sigma^+ \) potentials each support a single bound state. Due to the large density of bound vibrational states in the \( X^1\Sigma^+ \) potential, the singlet scattering length \( a_S \) is very sensitive to uncertainties in the potential. Indeed, deepening the reference potential by 1% for KH allows a new bound state to appear, causing \( a_S \) to pass through infinitely large values.

We may estimate the scattering length from effective range theory as

\[ a_{\text{eff}} = \frac{\hbar}{\sqrt{2\mu E_b}}. \]  

Table II. Scattering lengths in a.u. for \( a^3\Sigma^+ \) and \( X^1\Sigma^+ \) potentials. The entries marked hybrid were calculated employing potentials constructed in Ref. [5]. Other values were obtained with potentials uniformly scaled as \( U(R) \to \lambda U(R) \).

<table>
<thead>
<tr>
<th>Potential</th>
<th>( a^3\Sigma^+ )</th>
<th>( X^1\Sigma^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_S )</td>
<td>( a_T )</td>
<td>( a_S )</td>
</tr>
<tr>
<td>hybrid</td>
<td>43.2</td>
<td>42.3</td>
</tr>
<tr>
<td>+1%</td>
<td>44.3</td>
<td>35.2</td>
</tr>
<tr>
<td>−1%</td>
<td>42.3</td>
<td>34.3</td>
</tr>
<tr>
<td>hybrid</td>
<td>−9.77</td>
<td>−2.99</td>
</tr>
<tr>
<td>+1%</td>
<td>−83.6</td>
<td>41.9</td>
</tr>
</tbody>
</table>

where \( E_b \) is the binding energy of the last bound state. The resulting scattering lengths are presented in Table I; they are in fair agreement with the results of the direct calculations listed in Table II.

The total elastic cross section is expressed through the phase shifts as

\[ \sigma = \frac{4\pi}{k^2} \sum_{l=0} \sin^2 \delta_l, \]  

while the transport (momentum-transfer) cross section is calculated from

\[ \sigma_{\text{tr}} = \int \cos \theta (1 - \cos \theta) d\sigma, \]

\[ = \frac{4\pi}{k^2} \sum_{l=0} (l+1) \sin^2 (\delta_{l+1} - \delta_l). \]

The results are shown in Fig. 3. For ultracold temperatures, where s-wave scattering dominates, the total and the transport cross section are equal and are given by \( 4\pi a^2 \). The \( l=1 \) partial waves start contributing at the 1% level when energies of \( 8 \times 10^{-8} \) hartree are reached for NaH.

Assuming equal temperatures of the hydrogen and alkali-metal gases, we determine rate coefficients for elastic scattering as

\[ k_r = \sqrt{\frac{8}{\pi \mu \beta}} \int_0^{\beta E} (\beta E) \sigma(E) e^{-\beta E} d(\beta E), \]

where \( \beta = 1/(k_B T) \), \( E = h^2 k^2/(2\mu) \), and \( \mu \) is the reduced mass of the system. The results of the calculation are presented in Fig. 4. For temperatures below 1 mK, the rate coefficients are accurately approximated by

\[ k_r = 4\pi a^2 \sqrt{8/(\pi \mu \beta)}, \]

where \( a \) is the relevant scattering length.

In experiments on Bose-Einstein condensation [1], a cloud of hydrogen atoms is placed in a magnetic field, which traps spin-polarized low-field seeking atoms. The corresponding triplet scattering length [7] is \( a_T^{H-H} = 1.212a_0 \) (\( a_0 \): bohr radius). Our calculations demonstrate that the triplet

FIG. 2. \( X^1\Sigma^+ \) potentials for alkali hydrides [5].

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elastic cross sections for the sodium-hydrogen pair ($\sigma = 4 \pi a^2$) are 640 times larger than that for the hydrogen-hydrogen pair ($\sigma = 8 \pi a^2$). For the rubidium-hydrogen pair, this factor is 860. The efficiency of the kinetic-energy transfer to the hydrogen atom is proportional to the ratio of the mass of the hydrogen atom to the mass of the alkali-metal atom A. Accordingly we define as a measure of the thermalization efficiency of a single collision of a hydrogen atom with a buffer-gas atom the ratio

$$f = \frac{M_H \sigma(A-H)}{M_A \sigma(H-H)} = \frac{1}{2} \left( \frac{a_T^{A-H}}{a_T^{H-H}} \right)^2.$$

The calculated values are listed in Table III. We include $^7$Li for which the triplet scattering length [2] is $a_T = 65(5)a_0$. Due to its smaller mass, $^7$Li may be the best candidate for a buffer-gas cooling technique with an efficiency factor $f \approx 205$. Sodium has $f = 28$; K, Rb, and Cs are less efficient with $f \approx 10$.

Spin-flip collisions lead to trap losses. In the limit of small energies of relative motion, the corresponding cross section is estimated by $\sigma_{SF} = \pi(a_T - a_S)^2$ [8]. Using scattering lengths calculated with the reference potentials, we find for NaH $\sigma_{SF} = 8800$ a.u. Unfortunately, due to the large uncertainties in the singlet scattering lengths, no definite conclusions can be drawn about spin-flip losses, except that they are probably large if collisions are not in a pure triplet state.

In the hydrogen BEC experiment [1], the Ioffe-Pritchard magnetic trap is loaded with hydrogen atoms at $T \approx 250$ mK. Further cooling is achieved by contact of the atoms with a cryogenic wall. The large frequency of the Ly-\(\alpha\) transition prevents the use of optical cooling for hydrogen. As an alternative, an alkali-metal buffer gas could be introduced and laser cooled driving down the temperature of the hydrogen component through collisions with alkali atoms. At this active stage the temperature of the combined system could be potentially lowered to 50–100 \(\mu\)K (e.g., as in the recent MIT experiment [4] on sodium BEC). The critical

<table>
<thead>
<tr>
<th>A</th>
<th>$^7$Li</th>
<th>$^{23}$Na</th>
<th>$^{39}$K</th>
<th>$^{87}$Rb</th>
<th>$^{133}$Cs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>205</td>
<td>28</td>
<td>11</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

FIG. 3. Elastic (solid lines) and transport (dashed lines) cross sections for collision of spin-polarized alkali-metal atoms and hydrogen.

FIG. 4. Elastic (solid lines) and momentum-transfer (dashed lines) rate coefficients for collisions of spin-polarized alkali-metal atoms and hydrogen.
The temperature of the hydrogen BEC transition is 50 μK [1]. Therefore, depending on the densities, the BEC conditions for the hydrogen component could be achieved at the optical stage. In contrast, temperatures no lower than 200 μK have been obtained for Li [9]. Although Li (f = 205) is more efficient than Na (f = 28) at cooling H, the critical temperature cannot be reached by optical cooling using Li.

In the subsequent passive scheme both components could be evaporatively cooled simultaneously. The evaporative cooling of a hydrogen cloud is very slow due to the small H-H elastic cross sections. In the combined system with Na as the buffer gas, the sodium atoms would serve as a source of thermalization for hydrogen. The efficiency of thermalization is determined by both cross sections and the relative densities of two gases. Peak densities of sodium BEC with \( n_p = 1.5 \times 10^{14} \text{ cm}^{-3} \) comparable to densities of hydrogen [1] were recently reported [3] (similarly, Rb could also be used, although momentum transfer is less efficient). However, superfluidity properties of a sodium (or rubidium) condensate may limit the efficiency of sympathetic cooling of hydrogen at extremely low temperatures [10].

To summarize, an efficient pathway for achieving Bose-Einstein condensation of hydrogen through enhanced cooling by a buffer gas of sodium atoms seems feasible, due to large elastic cross sections for sodium-hydrogen collisions. We emphasize that it may be possible to optically cool the combined system to the critical conditions required for a hydrogen BEC.

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