Generalized Theory of Ion Impact Broadening in Magnetized Plasmas and Its Applications for Tokamaks

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A generalized semiclassical theory of ion impact broadening in high-temperature, magnetized plasmas is developed that is free from a shortcoming of the standard semiclassical theories of Stark broadening, which were intrinsically divergent at small impact parameters. The convergence of the present theory is achieved by taking into account, on equal footing, both the "dynamic" splitting of Stark sublevels, caused by one of the components of the ion microfield, and the Zeeman splitting. The results are applied to a novel spectroscopic method for local measurements of an effective charge in tokamaks.

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Introduction.—A new spectroscopic method for *local* measurements of an effective charge $Z_{\rm eff} = \sum Z_i^2 N_i / N_e$ of high-temperature plasmas (e.g., in tokamaks) was proposed [1] and recently implemented [2]. The idea of the method is the following. Under conditions typical of tokamaks ($N_e \sim 10^{13}$ cm^{-3}, $T_a \geq 10^2$ eV), Stark broadening of hydrogen spectral lines (SL) is controlled by *ion impact broadening (IIB)*. Then a resulting homogeneous Stark width $\gamma_s \propto \sum Z_i^2 N_i / \langle V_i \rangle$ is a linear function of $Z_{\rm eff}$ [the summation includes both the major plasma component (H^+ or D^+) and impurity ions].

A homogeneous width may be determined experimentally, using two different techniques. The first technique employs a saturation of an optical transition $n' - n$ in hydrogen by laser light with a small spectral width. Indeed, in this case, the observed fluorescence SL shape has the Voigt profile with a dispersive component of a halfwidth $\Gamma_B \approx (d_{12}E_0/\hbar)[(\gamma_{r,n}^{-1} + \gamma_{r,n'}^{-1})\gamma_{s,nn'}]^{1/2} \gg \gamma_{s,nn'}$, where $\gamma_{r,n}$ and $\gamma_{r,n'}$ are the radiative widths of the levels n and n', $\gamma_{s,nn'} \gg \gamma_{r,n} + \gamma_{r,n'}$, and E_0 is the laser amplitude. This significant enhancement of the width of the dispersive component makes it comparable to the Doppler width and allows us to extract it from the observed Voigt profile and thus allows us to measure $Z_{\rm eff}$.

The second technique is based on the Doppler-free two-photon excited fluorescence. A significant outburst of interest in applying this technique to measure various parameters of tokamak plasmas, including $Z_{\rm eff}$, was manifested this year [3]. A high quality laser spectrometer allowing for the realization of this diagnostic with an acceptable signal-to-noise ration has been recently developed and tested on a simulation plasma device [4].

However, whether or not either method of deducing $Z_{\rm eff}$ from the measured homogeneous Stark width will be of a broad practical use is contingent upon developing a *detailed, consistent theory of IIB,* as opposed to the rough estimates of IIB employed in [1–3]. This development constitutes the subject of the present paper.

Our theory of IIB has two distinctive features. First, we take into account a strong magnetic field $B \geq 1$ T, characteristic for tokamaks. Second, our theory of IIB is free from a shortcoming of the standard semiclassical theories of Stark broadening, which were intrinsically divergent at small impact parameters. The *convergence* of our theory is achieved in the spirit of paper [5].

We will show that practically all hydrogen SL *cannot* be employed for deducing $Z_{\rm eff}$ from the homogeneous Stark width. However, we have found a *unique phenomenon* of general theoretical interest that excludes the SL L_α from this rule and *indeed allows us to measure locally* $Z_{\rm eff}$.

Generalized ion impact broadening operator for hydrogen lines in a magnetic field.—The Hamiltonian of a hydrogen atom under the action of a static uniform magnetic field $\mathbf{B}$ and an ion-produced dynamic field $\mathbf{E}(t)$ may be written in the form

$$H \equiv H_0 + \mu_B \mathcal{L}\mathbf{B} - \boldsymbol{d}\mathbf{E}(t), \qquad (1)$$

where H_0 is an unperturbed Hamiltonian, μ_B is the Bohr magneton, and $\mathcal{L}$ and $\boldsymbol{d}$ are angular momentum and dipole moment operators, respectively. In the case where broadenings by both the ion and electron microfields of a plasma are impacted, the former dominates and therefore the latter may be disregarded [6]. Choose the axis Oz of the parabolic quantization along the magnetic field $\mathbf{B}$. Then the operator of the magnetic interaction $\mu_B \mathcal{L}\mathbf{B} = \mu_B L_z B$ is diagonal and can be allowed for exactly. As for the electric interaction $-\boldsymbol{d}\mathbf{E}(t)$ in the approximate theory of the IIB [6], it was treated in the second order of the time-dependent perturbation theory.

One of the central points of our theory is taking into account the interaction with a z component of the field $\mathbf{E}(t)$ on the same footing as the magnetic interaction. The physical idea behind our approach is that the entire operator $\mu_B L_z B - d_z E_z(t)$ (and not only its part $\mu_B L_z B$) is diagonal in any subspace of a fixed principal quantum number n in the parabolic quantization. Therefore, the z component of the ion field may be allowed for more accurately than in the standard IIB theory [6].

Consider a radiative transition between parabolic states $\alpha, \alpha', \ldots$ of an upper multiplet a and parabolic states

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$\beta, \beta', \ldots$ of a lower multiplet b. A standard formula for the IIB operator is (see, e.g., [6])

$$\Phi_{ab} = \sum_i \Phi^i_{ab},$$
$$\Phi^i_{ab} = N_i(1 + M_a/M_i)^{-1/2} \int_0^\infty v f(v)\, dv \times \int_0^\infty 2\pi\rho\, d\rho \{S_a S_b^* - 1\}_{\text{ang}}, \quad (2)$$

where N_i and M_i are a density and a mass of ions with a charge Z_i, respectively, and M_a is the atomic hydrogen mass. In distinction from the standard theory [6], we represent the scattering matrix S from (2) in the form

$$S = \exp\left[i \int_{-\infty}^{\infty} dt\, d_z E_z(t) \right] T \exp\left[i \int_{-\infty}^{\infty} dt\, Q^* \boldsymbol{d}_\perp \mathbf{E}_\perp(t) Q \right],$$
$$Q \equiv \exp\left\{ -i \left[H_0 t + \mu_B L_z B t - \int_{-\infty}^{t} dt'\, d_z E_z(t') \right] \right\}. \quad (3)$$

Here and below, unless specified to the contrary, the units are chosen such that $\hbar = 1$.

We break down the operator Φ_{ab} into two physically different parts: $\Phi_{ab} = (\Phi_{\text{ad}})_{ab} + (\Phi_{\text{na}})_{ab}$. The adiabatic part corresponds to using $(\Phi_{\text{ad}})_{ab}$ in (2) instead of S as the zero-order term of the Dyson expansion from (3)

$$S_{\text{ad}} = \exp\left[i \int_{-\infty}^{\infty} dt\, d_z E_z(t) \right], \quad (4)$$

that contains only the electric field component E_z parallel to the magnetic field $\mathbf{B}$.

The nonadiabatic part corresponds to using $(\Phi_{\text{na}})_{ab}$ in (2) instead of S as the second-order term of the Dyson expansion from (3) (the first-order term of the expansion vanishes after averaging). The nonadiabatic contribution that is quadratic with respect to $\mathbf{E}_\perp$ physically results from the virtual transitions between parabolic states induced by the electric-field components E_x, E_y orthogonal to the magnetic field $\mathbf{B}$.

Let us first calculate the nonadiabatic contribution $(\Phi^i_{na})_{ab}$ from only one sort of perturbing ions with a charge Z_i. Following paper [5] we obtain

$$\langle \alpha\beta | \Phi^i_{\text{na}} | \alpha'\beta' \rangle = -Z_i^2 N_i m_i g a_0^{-2} \left\{ \sum_{\alpha''} x_{\alpha\alpha''} x_{\alpha''\alpha'} \int_0^\infty dZ\, Z^{-1} C(\chi_{\alpha'}, |Y_{i\alpha}|, Z_\alpha) + \sum_{\beta''} [(\alpha, \alpha', \alpha'') \to (\beta, \beta', \beta'')] \right\}^*, \quad (5)$$

$$g \equiv (8\hbar^2/3m_e^2)(2\pi/T)^{1/2}, \quad m_i \equiv [M_i M_a/(M_i + M_a)]^{1/2}, \quad (6)$$

where the generalized broadening function $C(\chi, Y_i, Z)$ is

$$C_\pm(\chi, |Y_i|, Z) = \tfrac{3}{4} \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1} dx_2 \{ \varepsilon^{-1} \sin\varepsilon + (2x_1x_2 - 1)\varepsilon^{-3}(\sin\varepsilon - \varepsilon\cos\varepsilon)$$
$$+ [(1 - x_1x_2)\sigma_1^2 - (x_1 + x_2)\sigma_1\sigma_2]\, \varepsilon^{-5}(3\sin\varepsilon - 3\cos\varepsilon - \varepsilon^2\sin\varepsilon)\} [w(x_1)w(x_2)]^3 \exp[iZ(x_1 - x_2)],$$
$$\varepsilon \equiv (\sigma_1^2 + \sigma_1^2)^{1/2}, \quad \sigma_1 \equiv Y_i Z^{-1}[x_1 w(x_1) \pm x_2 w(x_2) + 1 \pm 1 - 2\chi], \sigma_2 \equiv Y_i Z^{-1}[w(x_1) \pm w(x_2)], \quad (7)$$
$$w(x) \equiv (1 + x^2)^{-1/2}.$$

Here the upper signs correspond to the nondiagonal elements ($\alpha' \neq \alpha$ or $\beta' \neq \beta$) of the xx operator in (5), and the lower signs correspond to the diagonal elements ($\alpha' = \alpha$ in the first term or $\beta' = \beta$ in the second term). Three pairs of dimensionless parameters χ_k, Y_k, Z_k ($k = a, b$) in (5) stand for

$$\chi_a \equiv (n_\alpha q_\alpha \delta_{\alpha\alpha'} - n_\beta q_\beta \delta_{\beta\beta'})/[n_\alpha(q_\alpha - q_{\alpha''})],$$
$$\chi_b \equiv (n_\alpha q_\alpha \delta_{\alpha\alpha'} - n_\beta q_\beta \delta_{\beta\beta'})/[n_\beta(q_\beta - q_{\beta''})],$$
$$q \equiv (n_1 - n_2); \quad Z_a \equiv \rho\mu_B B(m_\alpha - m_{\alpha''})/(\hbar v), \quad (8)$$
$$Z_b \equiv \rho\mu_B B(m_\beta - m_{\beta''})/(\hbar v);$$
$$|Y_{ia,b}| = 3n_{\alpha,\beta} Z_i \mu_B B/(2m_e v^2) \approx 0.160 n Z_i B(\text{T})/T(\text{eV}).$$

It is worthwhile emphasizing two points. First, our theory of the IIB is a *generalization* of the standard theory of the IIB [6]—it embraces the latter as the limiting case $Y \to 0$. Second, the integral of Eq. (5) *converges* for any finite Y. A physical reason for the convergency will be discussed below.

Shape of the L_α line.—The standard expression for the line shape reads [6]

$$I_{ab}(\omega) = -\pi^{-1} \text{Re} \sum_{\alpha\alpha'\beta\beta'} \langle\beta|\boldsymbol{d}|\alpha\rangle\langle\alpha'|\boldsymbol{d}|\beta'\rangle \times \langle\alpha\beta|G^{-1}|\alpha'\beta'\rangle, \quad (8)$$

where the operator G has the form

$$G = i(\omega - \omega_{ab}) + \Phi_{ab},$$
$$\omega_{ab} \equiv [(H_0 + \mu_B L_z B)_a - (H_0 + \mu_B L_z B)_b]/\hbar. \quad (9)$$

Figure 1 shows a scheme of Zeeman splitting of the hydrogen level $n = 2$ labeled by the parabolic quantum

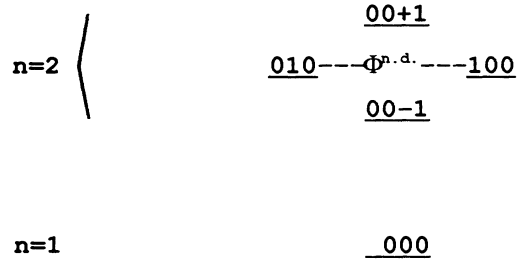

FIG. 1. Scheme of Zeeman splitting of the hydrogen level $n = 2$ labeled by parabolic quantum numbers (n_1, n_2, m). Degenerate states (010) and (100) are interconnected by a nondiagonal matrix element Φ^{nd} of the ion impact broadening operator.

numbers $(n_1 n_2 m)$. Degenerate states (010) and (100) are interconnected by a nondiagonal matrix element Φ^{nd} of the nonadiabatic part of the IIB operator $\Phi_{na} \propto xx$. Therefore, in accordance with paper [7] an inversion of the operator G in (8) results in the following structure of the SL L_α. The line breaks down into two isolated singlets I_σ^+, I_σ^- corresponding to the lateral σ components (001) → (000), (00 − 1) → (000) and a central π doublet I_π corresponding to the two merged subcomponents [(100), (010) → (000)].

Each of the two isolated lateral σ components has the Lorenzian shape of a HWHM

$$\gamma_\sigma^i = -\mathrm{Re}\langle 001|\Phi_{na}^i|001\rangle = (9g/2)a_-(0, |Y_i|), \quad (10)$$

$$a(\chi, |Y_i|) \equiv \int_0^\infty dZ\, Z^{-1}\, \mathrm{Re}C(\chi, |Y_i|, |Z|), \quad (11)$$

where suffix i reflects a dependence on the perturbing ion charge Z_i. The adiabatic contribution to the width of these components is zero.

The central π component has the Lorenzian shape with a more complicated formula for the HWHM

$$\gamma_\pi^i = \Gamma_\pi^i - |\beta_\pi^i|, \quad (12)$$

resulting not only from the diagonal elements

$$\Gamma_\pi^i = \Gamma_{\pi na}^i + \gamma_{\pi ad}^i, \quad (13)$$

$$\Gamma_{\pi na}^i \equiv -\mathrm{Re}\langle 100|\Phi_{na}^i|100\rangle = -\mathrm{Re}\langle 010|\Phi_{na}^i|010\rangle = (9g/2)a_-(1, |Y_i|), \quad (14)$$

$$\gamma_{\pi ad}^i \equiv -\mathrm{Re}\langle 100|\Phi_{ad}^i|100\rangle = -\mathrm{Re}\langle 010|\Phi_{ad}^i|010\rangle, \quad (15)$$

but also from the nondiagonal elements

$$\beta_\pi^i \equiv \mathrm{Re}\langle 100|\Phi_{na}^i|010\rangle = \mathrm{Re}\langle 010|\Phi_{na}^i|100\rangle = -(9g/2)a_+(0, |Y_i|) \quad (16)$$

(the adiabatic contribution to the nondiagonal elements is zero).

From formulas (10)–(16) it is clear that for every component of the SL the width γ^i consists of adiabatic and nonadiabatic parts: $\gamma^i = \gamma_{na}^i + \gamma_{ad}^i$. The central point is that the actual dependence of γ_{na}^i on the parameters of a particular sort of perturbing ions $\gamma_{na}^i \propto Z_i^2 N_i m_i^{1/2} a(Y(Z_i))$ is much more complicated than it was assumed in papers [1,2]. Indeed, the first assumption in [1,2] was that, for any kind of perturbing ions, the reduced mass m_i of the atom-ion pair is equal to the proton mass m_p. While this is true for heavy impurity ions, the assumption is not valid for the primary perturbers—for protons: for the latter case $m_i = m_p/2$. However, this circumstance might be easily accounted for if we would disregard for a moment the dependence of the broadening function a on Z_i. Then we would obtain $\gamma_{na} = \sum_i \gamma_{na}^i \propto (Z_{eff} - 1 + 2^{-1/2})$ instead of $\gamma_{na} \propto Z_{eff}$, so that it would still be possible to deduce Z_{eff} from the experimental width.

Regrettably, the situation is much worse. The most important assumption in [1,2] was that all other factors beyond the product $Z_i N_i m_i^{1/2}$ could be disregarded while performing the summation over i. This assumption was based on the fact that, in the standard theory of IIB, the broadening function a_{stand} (obtained via an artificial cutoff of a diverging integral) has only a very weak logarithmic dependence on Z_i. In reality, the broadening function $a(Y(Z_i))$ that we obtained above, via the convergent integration, has a very strong dependence on Z_i and *cannot be factored out of the summation over i*.

This important point is illustrated by Fig. 2 where, for the σ components of the SL L_α chosen as an example, $\mathrm{Re}C_-(Y(Z_i), Z)$ is plotted versus the dimensionless impact parameter Z for three different values of the perturbing ion charge: $Z_i = 1$, 6, and 26. The abscissa scale is logarithmic so that the function $a_-(Y(Z_i))$ defined in (11) is equal to the area under the curve in Fig. 2, corresponding to a particular value of Z_i. It is clearly

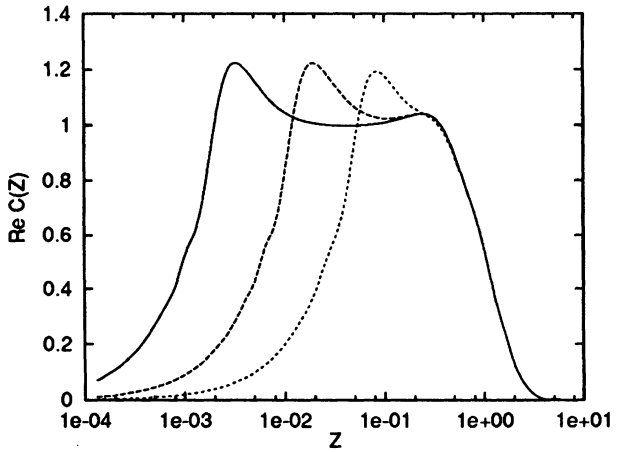

FIG. 2. The broadening function $\mathrm{Re}C_-(Y(Z_i), Z)$ versus the dimensionless impact parameter $Z \equiv \rho\mu_B B(m_\alpha - m_{\alpha''})/\hbar v$ for three different values of the perturbing ion charge: $Z_i = 1$ (solid curve); $Z_i = 6$ (dashed curve); and $Z_i = 26$ (dotted curve). The calculation is made for the σ components of the L_α line under plasma parameters $N_e = 10^{13}$ cm^{-3}, $T = 100$ eV, $B = 2$ T.

seen that those areas [and, respectively, the functions $a_-(Y(Z_i))$] differ drastically for various ion charges Z_i.

Therefore, the nonadiabatic part $\gamma_{\rm na}$ of the homogeneous Stark width γ_s *cannot be expressed as a function of* $Z_{\rm eff}$. In other words, the same width γ_s may correspond to significantly different values $Z_{\rm eff}$.

This conclusion is based on the nature of the broadening function $a(Y(Z_i))$ rather than on the particular SL chosen and therefore has a general character. Practically all hydrogen SL *cannot* be employed for deducing $Z_{\rm eff}$ from the homogeneous Stark width. Particularly, $Z_{\rm eff}$ inferred in [2] by using the SL H_α is regrettably irrelevant.

Fortunately, we have found one exception to this rule. The nonadiabatic contribution to the width of the central π component of the SL L_α in accordance with (12)–(16) has the form $\gamma^i_{\pi{\rm na}} = \Gamma^i_{\pi{\rm na}} - |\beta^i_\pi| = (9g/2)[a_-(1,|Y_i|) - a_+(0,|Y_i|)]$. It turns out that $[a_-(1,|Y_i|) - a_+(0,|Y_i|)] \sim 10^{-2}a(1,|Y_i|)$, so that $\gamma^i_{\pi{\rm na}}$ is negligibly small compared to the adiabatic contribution $\gamma^i_{\pi{\rm ad}}$. Actually, this is a result of a mutual cancellation of diagonal and nondiagonal matrix elements of the xx operator as manifested equally by modulus coefficients $9g/2$ in front of the a functions in (14) and (16).

Therefore, the impact width of the L_α central component reduces to the adiabatic contribution $\gamma^i_{\pi{\rm ad}}$. The latter was calculated using the "old" adiabatic theory of impact broadening [6] and given by formulas (53)–(56) in [5]. From those formulas we obtain

$$\gamma^i_{\pi{\rm ad}} = Z_i^2 N_i \gamma_0 / N_e\,,$$
$$\gamma_0 \equiv 72(\hbar/m_e)^2 N_e (2\pi m_p/T)^{1/2} I(R)$$
$$\approx 2.45\times 10^{-4} N_e({\rm cm}^{-3})[T({\rm eV})]^{-1/2} I(R)\,({\rm s}^{-1})\,;$$
$$I(R) \approx 0.209 + 6^{-1}\ln R\,, \qquad (17)$$
$$R \equiv (m_e V_p/6\hbar Z_i)[T_e/4\pi e^2 Z_{\rm eff} N_e]^{1/2}$$
$$\approx 1.1\times 10^8 T({\rm eV})\,[Z^3_{\rm eff} N_e({\rm cm}^{-3})]^{-1/2}\,.$$

A weak logarithmic dependence of γ_0 on Z_i in (17) allows us to factor γ_0 out of summation over i and to express the impact width of the central π component of the L_α as a function of $Z_{\rm eff}$:

$$\gamma^\pi_s = \sum_i \gamma^i_{\pi{\rm ad}} = (Z_{\rm eff} - 1 + 2^{-1/2})\gamma_0\,. \qquad (18)$$

We have developed a theory of IIB that is intrinsically *convergent* and, therefore, does not require an artificial, fuzzy-determined cutoff as the standard impact theories [1,2,6] did. From the physical point of view, the convergency was achieved using the break of the spherical symmetry of the unperturbed Hamiltonian down to the axial symmetry (due to the interaction with the static magnetic field) and treating the projection of the dynamic electric field onto the symmetry axis "preferentially"—more accurately than the electric field components orthogonal to the axis. The axial component of the dynamic electric field, being taken into account exactly (in all the orders of the Dyson expansion), effectively brought up some average Stark splitting of energy levels. The latter made contributions of small impact parameters into the width finite.

Further, in analyzing the impact widths of components of the Zeeman triplet for various hydrogen lines, we have found a "gem." For just one component of only one hydrogen SL L_α, the impact width is a purely adiabatic effect that was calculated by means of the old adiabatic theory of broadening. Let us recall that the old adiabatic theory is a *classical* model of a phase modulation of an (atomic) oscillator [6]. So what we have found means that the impact width of the L_α central component is described *classically*. In distribution from the usual expectation that quantum mechanics reduces to classical mechanics as the principal quantum number $n \to \infty$, in our case, the reduction to the classical result occurs for $n = 2$ (and, moreover, only for $n = 2$). This is an amazing result of general theoretical interest.

From the practical point of view, we have shown that, only due to this unique phenomenon, it is indeed possible to measure locally an effective charge $Z_{\rm eff}$ in tokamaks by means of the laser-induced fluorescence of the central (π) component of the SL L_α. There are two experimental techniques for these measurements as we mentioned. Let us estimate a laser field required for the first technique that will employ a saturation of the L_α transition. Under conditions typical of tokamak edge plasmas ($N_e = 10^{13}$ cm^{-3}, $T = 10^2$ eV) for $Z_{\rm eff} = 4$ the homogeneous width from (18) is $\gamma_s = 1.2\times 10^9$ s^{-1}. To observe the Voigt profile (with equal dispersive) and the Doppler components it should be feasible that it be used as a laser field $E_0 \sim 10^2$ kV/cm. Even a slightly higher laser field has already been used in experiment [4] that is considered as a practical basis for the second technique—for the Doppler-free two-photon excited fluorescence of the L_α line.

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