Undergraduate Thesis

Initial Search Results for Dark Matter Using the GPS.DM Observatory

University of Nevada, Reno

Conner Dailey
Advisors: Dr. Geoffrey Blewitt, Dr. Andrei Derevianko

Abstract

One of the most prevalent mysteries in astrophysics is that most of the matter in the universe is not luminous. The GPS.DM Observatory is committed to probing the universe for exotic dark matter (DM) physics using the Global Positioning System (GPS) satellite constellation. The current strategy for achieving this involves utilizing a search through GPS time series data for signatures of DM in the form of topological defects (TDs) passing through the Earth. Models predict that DM interactions with the satellite's atomic clocks may produce a difference in the relative atomic clock timekeeping between two satellites. TD thin walls can then be shown to create a predictable signal in the GPS time series data. Using expressions for the orbits of the satellites, and a derivation of the direction from which we expect to find these DM events, a search was conducted for the signatures left by these walls. This paper describes how the initial search for TD thin walls was conducted. Though we found no concrete evidence for TD thin walls with timing differences higher than 0.5ns, several candidate events were found that may correspond to other types of TDs.

Acknowledgements

I am thankful for the opportunity to work with the GPS.DM observatory. I learned so much from this experience and I look forward to applying what I learned to my future career. I would like to thank Dr. Andrei Derevianko and Dr. Geoffrey Blewitt for their great mentorship to me and my colleagues. I am thankful to Dr. Ben Roberts who was always there to answer questions for me and help with my research. I would also like to thank the Office of Undergraduate Research who supported me through the Undergraduate Research Award, and the National Science Foundation who supported me through a student grant.

Contents

1 Introduction ... 1
2 Keplerian Elements ... 2
3 Velocity of the Earth Through the Galaxy ... 3
3.1 Solar System Barycentric Velocity ... 4
3.2 Annual Variation of the Earth's Velocity ... 5
4 Atomic Clocks and DM Interactions ... 7
5 Expected Signals from a DM Event ... 8
5.1 The Thin Wall Model ... 9
5.2 Unknown Thin Wall Parameters ... 13
5.3 Characteristics of the DM Search Function ... 15
6 Search Results ... 17
7 Conclusion ... 18
References ... 19
Appendix A: Orbital Elements Conversion Equations and Mathematica Code ... 20
Appendix B: Mathematica Code for DM Thin Wall Signal Search ... 25
Appendix C: Search Results with the Highest Matching Signals ... 31

List of Figures

Figure 2.1: Definition of angles used in Keplerian Elements.. 3
Figure 3.1: Plot of the direction of the Earth's velocity vector in the ECI frame over one year... 6
Figure 3.2: Path of the Earth's velocity vector on the sky.. 7
Figure 5.1: Plot of the step functions expected from time bias differences seen in DM wall interactions with atomic clocks.. 9
Figure 5.2: Plot of a 30-satellite constellation bias signal. 10
Figure 5.3: Representation of a DM Wall event with the thin wall model. 10
Figure 5.4: Discrete signal corresponding to Figure 5.2... 11
Figure 5.5: Differenced signal corresponding to Figure 5.3. 13
Figure 5.6: Example of a thin TD wall crossing the Earth. 14
Figure 5.7: Probability density function of velocities and normal velocities of thin walls. 14
Figure 5.8: Sample Data of a random day shows frequent vertical lines 16
Figure 6.1: Demonstration of the difference in signal fit 17

1 Introduction

The GPS satellite constellation consists of 32 satellites in medium Earth orbit. Each satellite carries on board an atomic clock that keeps time extremely accurately, which can be monitored using data acquired by a network of ground receivers, that is used primarily for global positioning. The best data from the constellation has been taken since May 2001, when the U.S. military turned off selective availability (intentional dithering of the satellite signals). NASA's Jet Propulsion Laboratory (JPL) provides data on the precise timekeeping of each satellite for most of this time. Atomic clocks are well suited for detecting minute physical effects because the best atomic clocks can keep a fractional inaccuracy of 10^{-18} [1]. If DM interacts with these clocks, the constellation can potentially serve as a 50,000-km aperture DM detector. The goal of the GPS.DM Observatory is to search the GPS time series data for signatures of DM using models of TDs. TDs come in a few different dimensional forms, monopoles (0D), strings (1D), and domain walls (2D) [1]. One specific model of TDs is the "thin wall" model, which is the model being tested in this paper. Thin walls are domain walls which are assumed to be large in comparison to the Earth and have a thickness that is not resolvable with the GPS constellation. DM may come in the form of these thin walls, which will affect the time data in a predictable way. Before we can search for this signature however, there are several important steps to complete first.

To better understand and predict the positions of the GPS satellites in the GPS.DM Observatory, we decompose the orbits into Keplerian elements that can be used to find the approximate position and velocity of the satellites at any time. The six Keplerian elements describe any ellipse in 3D space, with the assumption that the orbit ellipse is "Keplerian", with no forces acting on the satellite other than gravity from a point source. For the intents and purposes of the GPS.DM Observatory, the GPS satellites are approximately Keplerian and can be modeled as such for our analyses. These approximate orbits of the satellites will be used to help derive velocity distributions, and help with the future Bayesian analysis of the data.

Another important step is knowing to which direction in the Earth Centered Inertial (ECI) frame the Earth's overall galactic velocity vector is pointing with respect to the galactic reference frame. The Earth, the solar system, and even the galaxy is in constant motion, which can be difficult to model completely. This paper shows our mathematical approach to finding the unit vector of the Earth's galactic velocity in the ECI frame, and how it changes annually due to the Earth's orbit.

It is in the interest of our group to find signatures of DM using models of how it might interact with atomic clock timekeeping. Using these models, there are many unknown parameters that must be considered, such as the magnitude of a signal, the exact time a signal occurs, the velocity of the DM associated with that signal, and the direction from which a signal might come. A complete Bayesian search method over these parameters would involve lengthy integrals that would demand substantial computing power. My objective was to run a search for possible events that would require less computing power and would search for events without complete Bayesian analysis. The results of this search found a few potential events that can now be analyzed for validity.

2 Keplerian Elements

To model a satellite's Keplerian orbit in three dimensions, one first assumes that gravity is the only force acting on the system and that the Earth is a point source (equivalent to a spherically symmetric Earth). Together with Newton's laws of motion and gravitation, the differential equation of orbital motion can be derived [2]. The force of gravity is F_g, the mass of the satellite is m, the mass of the Earth is M, the position vector from the center of the Earth to the satellite is $\boldsymbol{r}$ and r being the magnitude of that vector, and G is the gravitational constant.

$$F_g = m\ddot{\boldsymbol{r}} = -\frac{GMm}{r^3}\boldsymbol{r} \tag{2.1}$$

Attempting to solve this differential equation leads to elliptic integrals, which are very difficult to solve in closed form, so instead a model of the orbit can be used that already takes the elliptic properties into account. Instead of using position (x, y, z) and velocity $(\dot{x}, \dot{y}, \dot{z})$, the six Keplerian Elements are used:

Orbital Parameter	**Symbol**
Semi-Major Axis	a
Eccentricity	e
Inclination	i
Longitude of the Ascending Node	Ω
Argument of Periapsis	ω
Mean Anomaly	M

Table 2.1: *Names of the orbital elements.*

The semi-major axis sets the scale factor of the ellipse and is defined as half of the largest diameter of the ellipse. The eccentricity defines how elliptical the shape is, zero being perfectly circular, one being parabolic, and anywhere in between being elliptic. Most satellite eccentricities are very near zero, corresponding to nearly circular orbits. The inclination is the angle from the equatorial plane to the plane of the ellipse. The longitude of the ascending node is the angle from the ECI x-axis to the ascending node, which is the point the satellite will cross the equatorial plane and be traveling in the positive z direction (where z is normal to the equatorial plane, pointing toward the north pole). The argument of periapsis is the angle from the ascending node to the lowest point in the orbit. Finally, the true anomaly is an angle that describes where in the orbit the satellite is at any given time, and is the only element that changes in time ideally [2].

These elements are visually defined by Figure 2.1. The true anomaly can be computed by knowing the mean anomaly, which is the true anomaly for a circular orbit of the same Semi-Major Axis.

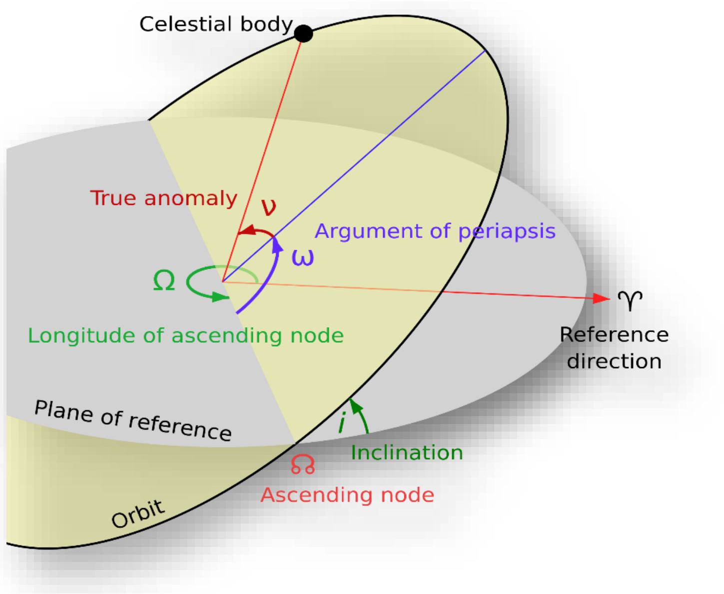


Figure 2.1: *Definition of angles used in Keplerian Elements. The plane of reference of the Earth is the equatorial plane, and the Reference Direction is the ECI x-axis. Image from [3].*

If the initial velocity and position of a satellite in 3D space are given, then the six Keplerian Elements that define an orbital ellipse can be found using algorithms produced from Newtonian dynamics and trigonometry. Then the mean anomaly can be time-stepped a desired amount to find the new Keplerian Elements in a different time, then converted back to find the position and velocity once more. All the equations for these conversions are given in Appendix A.

$$(x_0, y_0, z_0), (\dot{x}_0, \dot{y}_0, \dot{z}_0) \rightarrow (a, e, i, \Omega, \omega, M) \rightarrow (a, e, i, \Omega, \omega, M') \rightarrow (x, y, z), (\dot{x}, \dot{y}, \dot{z}) \quad (2.2)$$

Using this method, just from a set of initial conditions, an expression for a satellite's orbit can be produced, allowing for the ability to find its position and velocity at any time increment desired. However, due to the small non-Keplerian nature of the satellites, these calculated positions and velocities will get increasingly inaccurate as the time step gets large, though this inaccuracy is not significant for our purposes.

3 Velocity of the Earth Through the Galaxy

It is an important preliminary step in this research to determine where precisely, from the Earth's perspective, the velocity of our total motion in the galaxy is pointed. This direction will be the most probable for a DM event as explained in Section 5.2, as we can think of a "wind" of DM constituents from this direction. There are several velocities to consider for finding this wind direction. The first is the galactic orbital velocity, or the tangential velocity of our solar system's orbit around the galactic center, which is ~200km/s. The second is the "peculiar motion" or the velocity that accounts for our solar system's apparent oscillatory motion above and below the galactic disk. These oscillations happen with a period of about 70 million years [4]. The measured current magnitude of this peculiar motion is ~10km/s. The third part is the Earth's orbital motion around the sun, which is ~30km/s on average, but the direction of this motion rotates 360° throughout the year. The last thing to consider is the orbital motion of the GPS satellites themselves around the Earth. This motion is only ~4km/s, which can be considered negligible compared to the other velocity contributions.

3.1 Solar System Barycentric Velocity

To find the direction of our galactic orbital motion, the motion vectors given in galactic coordinates must be converted to the ECI frame. The galactic reference frame is defined as a right-handed coordinate system, with the vector that is parallel to our galactic orbital velocity in the direction of the galactic y-axis and the galactic center is the direction of the x-axis. The rotation matrix from the ECI frame to the galactic frame is given by [5]:

$$M_{ECI\to GAL} = \begin{pmatrix} -0.05465 & -0.87284 & -0.48492 \\ 0.49405 & -0.44567 & 0.74651 \\ -0.86771 & -0.19877 & 0.45559 \end{pmatrix} \tag{3.1}$$

Taking the inverse of this matrix:

$$M_{GAL\to ECI} = \begin{pmatrix} -0.05465 & 0.49405 & -0.86771 \\ -0.87284 & -0.44567 & -0.19877 \\ -0.48492 & 0.74651 & 0.45559 \end{pmatrix} \tag{3.2}$$

The total galactic orbital velocity direction of the solar system is the result of the tangential orbit motion, and the peculiar motion, both vectors in the galactic coordinate system are given by [6]. Normalizing the final product gives the unit vector.

$$\begin{pmatrix} 0\ \mathrm{km/s} \\ 220\ \mathrm{km/s} \\ 0\ \mathrm{km/s} \end{pmatrix} + \begin{pmatrix} 10.00\ \mathrm{km/s} \\ 5.23\ \mathrm{km/s} \\ 7.17\ \mathrm{km/s} \end{pmatrix} = \begin{pmatrix} 10.00\ \mathrm{km/s} \\ 225.23\ \mathrm{km/s} \\ 7.17\ \mathrm{km/s} \end{pmatrix} \to \begin{pmatrix} 0.04433 \\ 0.99851 \\ 0.03179 \end{pmatrix} \tag{3.3}$$

Thus, the conversion of galactic velocity direction to ECI direction is:

$$\begin{pmatrix} -0.05465 & 0.49405 & -0.86771 \\ -0.87284 & -0.44567 & -0.19877 \\ -0.48492 & 0.74651 & 0.45559 \end{pmatrix} \begin{pmatrix} 0.04433 \\ 0.99851 \\ 0.03179 \end{pmatrix} = \begin{pmatrix} 0.46332 \\ -0.49003 \\ 0.73838 \end{pmatrix} \tag{3.4}$$

Converting this unit vector into right ascension and declination:

$$\alpha = 313.40° \; \delta = 47.594° \tag{3.5}$$

This direction represents the velocity direction of the solar system barycenter through the galaxy. The Earth moves this vector around slightly due to its orbit around the sun. The Earth's orbit is inclined by sixty degrees to the galactic plane, so its solar system orbital velocity vector has a significant perpendicular component relative to the velocity vector of the solar system most of the year.

3.2 Annual Variation of the Earth's Velocity

To see how this direction changes over the year, the average Keplerian elements of the Earth in the solar system frame can generate average velocity vectors for the whole year. The elements can be found from the JPL Horizons web interface [7]. Then the mean anomaly can be found at different times during the year to estimate the velocity vector.

$$(a, e, i, \Omega, \omega, M\,) \rightarrow \left(1.49 * 10^{11}\, m, 0.017, 0°, 349°, 103°, M(t)\right) \rightarrow (\dot{x}(t), \dot{y}(t), \dot{z}(t)) \tag{3.6}$$

The velocity found in this conversion will be in the solar system ecliptic frame, so we need to convert it from this frame to the ECI frame, which is just a rotation along the x-axis by the 23.5 degree obliquity of the Earth's equator to the ecliptic plane, defined by Equation 3.7. Then we can convert from ECI to the galactic frame using the conversion matrix from Equation 3.1 and add it to the galactic orbital vector and peculiar motion.

$$R1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \tag{3.7}$$

$$\begin{pmatrix} \dot{x}_e(t) \\ \dot{y}_e(t) \\ \dot{z}_e(t) \end{pmatrix}_{Galactic} = M_{ECI \rightarrow GAL} \cdot R1(23.5°) \cdot \begin{pmatrix} \dot{x}_e(t) \\ \dot{y}_e(t) \\ \dot{z}_e(t) \end{pmatrix}_{Ecliptic} \tag{3.8}$$

$$\begin{pmatrix} 0\, km\, s^{-1} \\ 220.\, km\, s^{-1} \\ 0\, km\, s^{-1} \end{pmatrix} + \begin{pmatrix} 10.00\, km\, s^{-1} \\ 5.23\, km\, s^{-1} \\ 7.17\, km\, s^{-1} \end{pmatrix} + \begin{pmatrix} \dot{x}_e(t) \\ \dot{y}_e(t) \\ \dot{z}_e(t) \end{pmatrix}_{Galactic} = \begin{pmatrix} \dot{x}_g(t) \\ \dot{y}_g(t) \\ \dot{z}_g(t) \end{pmatrix} = v_g(t)_{Galactic} \tag{3.9}$$

$$v_g(t)_{ECI} = M_{GAL \rightarrow ECI} \cdot v_g(t)_{Galactic} \tag{3.10}$$

Once $v_g(t)_{ECI}$ has been normalized and converted into right ascension and declination, it is straight-forward to plot this over time. Figure 3.1 represents this plot with day intervals between the plotted points. The dot in the middle of the graph represents the solar system barycentric velocity direction derived earlier. The velocity of the Earth through the galaxy on average is approximately the velocity of the solar system, but at any given time is ~8 degrees of arc away from it. Figure 3.2 represents the annual variation projected on the sky.

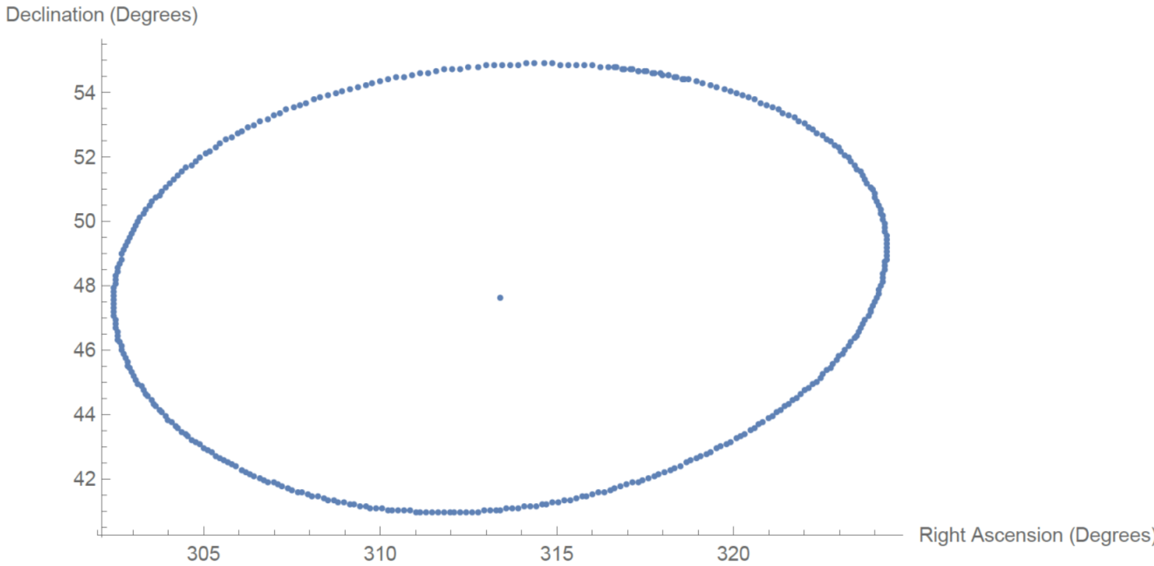


Figure 3.1: *Plot of the direction of the Earth's velocity vector in the ECI frame over one year.*

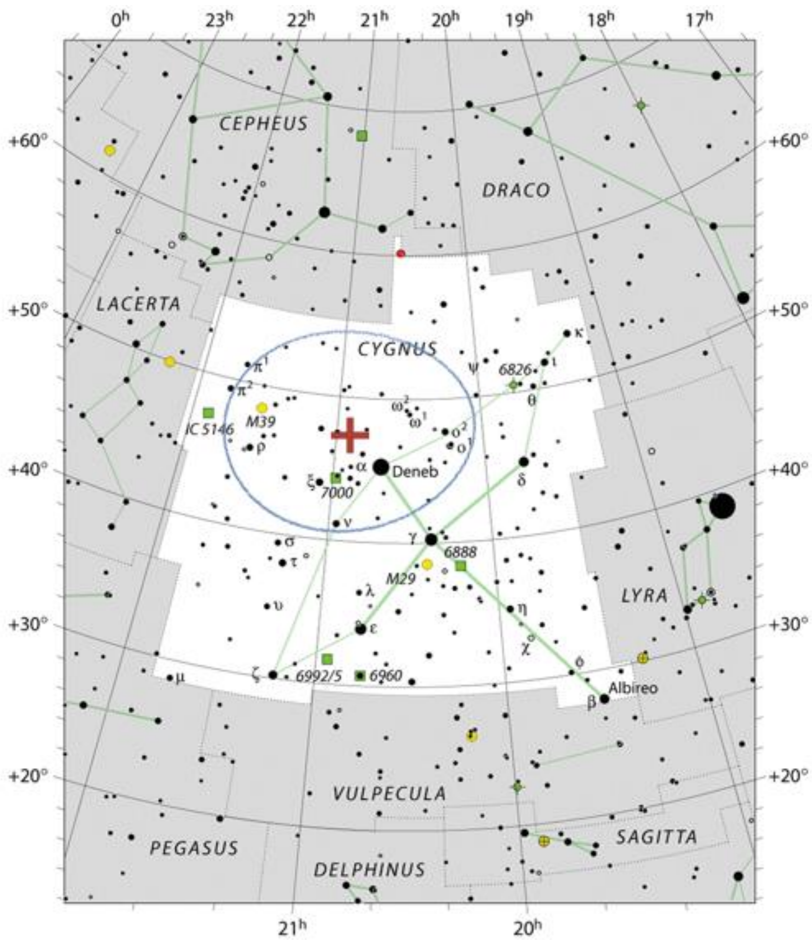


Figure 3.2: *Path of the Earth's velocity vector on the sky, the average represented by the red crosshairs. Constellation image from [8].*

The exact direction of our velocity through the galaxy is where we expect to have the highest probability of a DM event. Since this direction changes noticeably throughout the year, if we see DM events frequently enough to resolve the velocity probability distribution, we can check to see if the directional changes match that of the distribution of events. This could aid in the validity of DM event discovery and helps exclude false positives.

4 Atomic Clocks and DM Interactions

GPS satellites have become an ever-present part of daily life, from cell phone positioning to continental movement measurements. They work by keeping precise time using atomic clocks, which are onboard every satellite. They are designed to enable positioning of a user on the Earth by timing accurately how long it takes a signal to reach a user, and with four or more of these time measurements a unique position estimate can be found. For the purposes of this research however, we are not primarily interested in positioning, but rather in the precise time measurements kept by the satellites themselves.

Rubidium atoms are used in the most accurate GPS atomic clocks and are the most common in today's satellites. In an atomic clock, Rubidium atoms are used to stabilize a crystal oscillator which "ticks the clock." The precise time-keeping of atomic clocks is performed by the continual observation of a ground state hyperfine transition in ^{87}Rb. This hyperfine spitting is due to the coupling of the magnetic dipole of the outermost valence electron with the nucleus. This specific transition is chosen because it is unaffected by external magnetic fields and is very stable. The clocks work on a feedback loop, where a crystal oscillator has an initial frequency that is used to keep the time, but this frequency may drift over time due to environmental conditions to which crystal oscillators are very sensitive. The microwave frequency that corresponds to the hyperfine transition energy in Rubidium is radiated on a vapor of the atoms. The frequency of the light radiated on the atoms is then adjusted for maximum absorption and minimum transmission, to keep the frequency as accurate to the hyperfine transition as possible. This frequency is used to correct the frequency of the crystal oscillator, which has a known ratio with the Rb frequency [9]. Rubidium atomic clocks are the most common on the GPS satellites, and are the most stable over our period of interest (up to several minutes), which is why we look exclusively at these satellites in our analyses. The accuracy behind this time-keeping method implies extreme sensitivity to changes in fundamental constants such as the fine structure constant and the electron and proton masses.

The theory behind this experiment hypothesizes a Standard Model interaction between TDs and matter. For TDs to be macroscopic in comparison to the GPS constellation, the TDs must be composed of ultralight fields. TDs are the result of rapid cooling in the early universe, where these ultralight fields experienced symmetry-breaking. The Standard Model TD interaction predicts that small changes can be seen in fundamental constants [1].

$$m_{e,p}^{\mathrm{eff}} = m_{e,p}\left(1 + \frac{\phi^2}{\Lambda_{e,p}^2}\right); \quad \alpha^{\mathrm{eff}} = \frac{\alpha}{1 - \phi^2/\Lambda_\gamma^2} \tag{4.1}$$

Equation 4.1 [1] describes how these changes to three fundamental constants, the electron and proton masses and the fine structure constant, change due to these interactions. The dark matter field is denoted by ϕ and the effective energy scale is Λ (which is equivalent to $1/\sqrt{\Gamma}$ where Γ is the coupling strength). Since we do not know how DM TDs interact with other fields, we do not know whether they will slow down atomic clocks, or speed them up, so we must allow for both possibilities. As these defects pass through the GPS constellation, a signal corresponding to differences in the satellite timekeeping can be predicted.

5 Expected Signals from a DM Event

To form a model of the signal we expect to find from a DM event, we must first find a way to be able to see differences in satellite timekeeping. The GPS data is given at a specific instance for each satellite as a clock bias, that is the difference between the satellite clock time and a chosen reference clock. The data we have from JPL is generated every 30-s, or one epoch, with JPL's reference clock typically being the US Naval Observatory's H-maser master clock. We take these clock bias data, then select the newest satellite as a new reference clock. By transforming the reference clock, we ensure the data are all related to the Rb frequency standard, thus avoiding the problem that different frequency standards may interact differently with DM. It is then easy to see differences accumulate as satellites are affected by this altered timekeeping of Rb clocks caused by DM.

5.1 The Thin Wall Model

We assume a TD model of "thin walls" as a starting point in this research. This means that the domain wall is so thin as to affect each satellite in less than the 30-s resolution we have, and each clock that lies in the plane of the wall is affected simultaneously. The DM wall signal we then expect is a sequence of clock bias step functions that happen in a predictable order, demonstrated by Figure 5.1. If a DM event first passes through some satellite, that satellite's clock would change to a certain bias relative to the reference clock, until it then passes through the reference clock and then bias between them would return to zero. If the reference clock is hit first, it will appear that the satellite bias would suddenly change in the opposite direction until the satellite is hit. If we pick a reference clock that is on the Earth, and a DM Wall comes from some directional vector, then the satellites that have a positive projection on that vector would be affected first and jump to a bias until the wall had then passed through the reference clock. Then the wall would affect the satellites with a negative projection in the opposite direction. This succession of affected satellites is visualized by Figure 5.1, 5.2 and 5.3. Due to the 30 second resolution of the data that is given for the GPS time series, the signal we are searching for has a discrete matrix of points described by Figure 5.4.

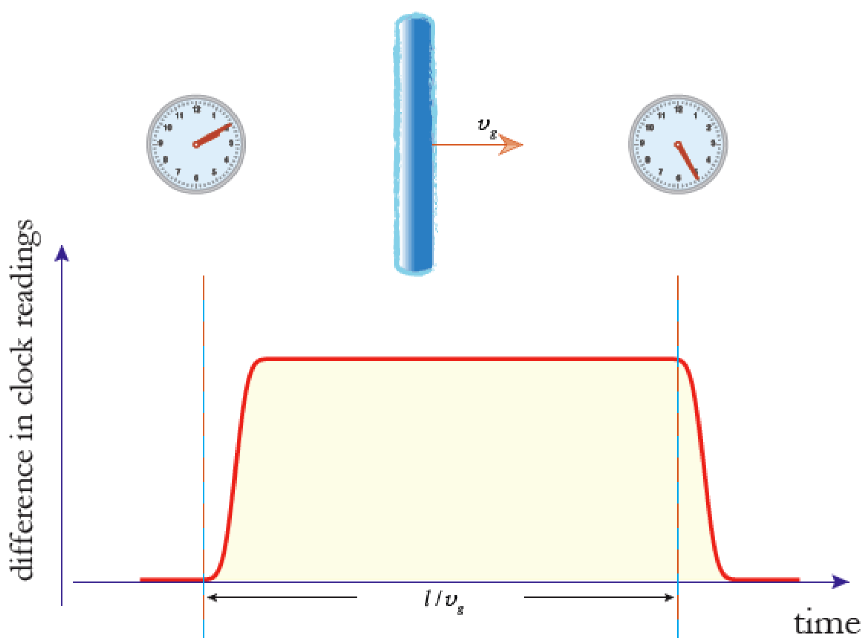


Figure 5.1: *Plot of the step functions expected from time bias differences seen in DM wall interactions with atomic clocks. The first clock is a random satellite clock, while then second one is the reference clock. There is about 20 of these step functions to keep track of as there is about 20 Rubidium satellites. [1]*

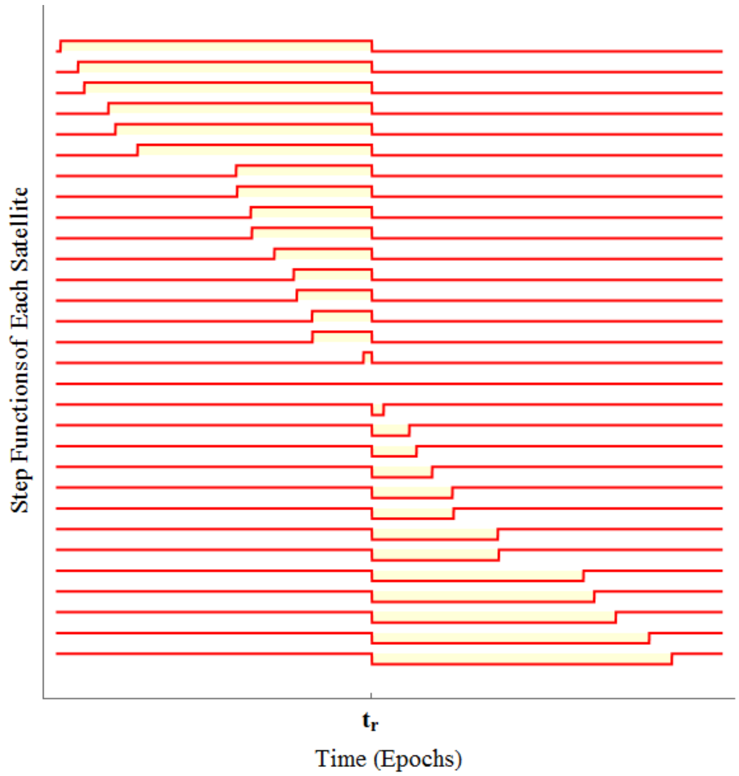


Figure 5.2: *Plot of a 30-satellite constellation bias signal ordered by the time the satellite is first crossed by the wall, where t_r is the time the reference clock is crossed.*

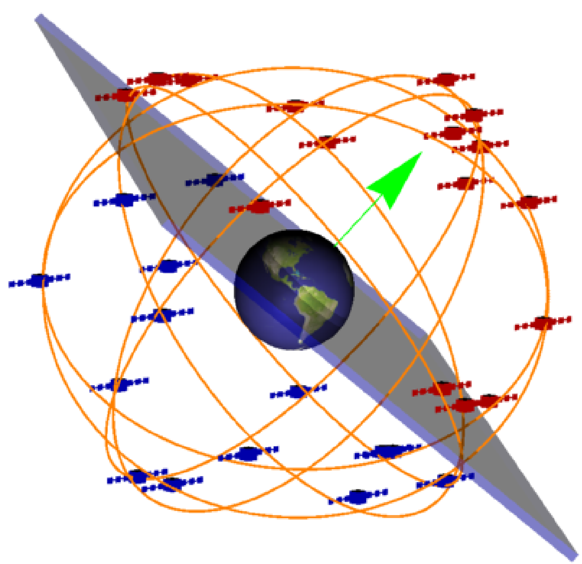


Figure 5.3: *Representation of a DM Wall event with the thin wall model.*

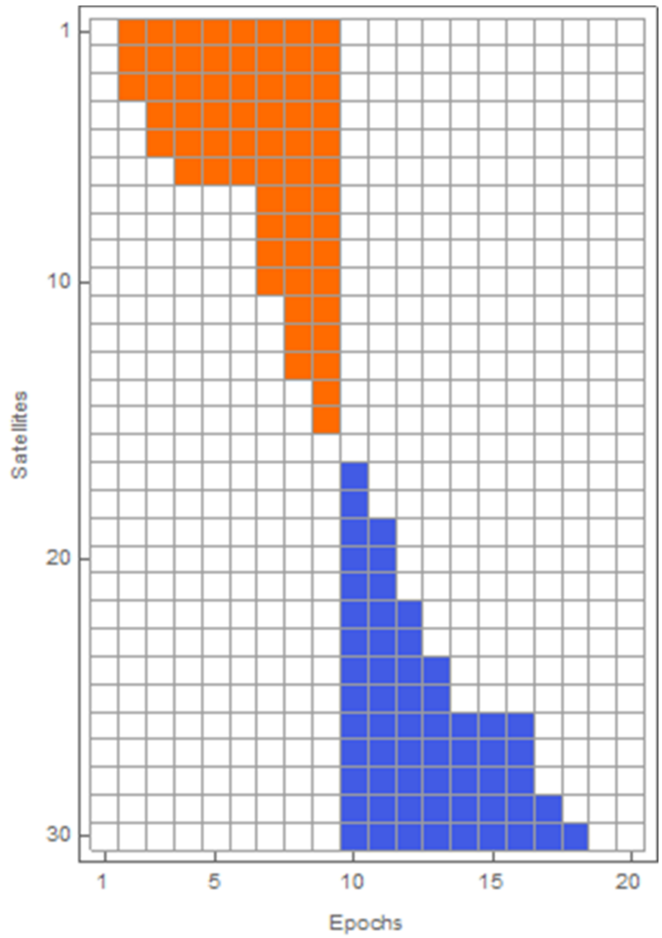


Figure 5.4: *Discrete signal corresponding to Figure 5.2. A positive bias is shown in orange, while a negative bias is shown in blue.*

The functional form of Figure 5.4 is given in Equation 5.1. This depends on quite a few different parameters.

$$F_i(t, t_i, t_r, h) = \begin{cases} h & t_i \le t < t_r \\ -h & t_i > t \ge t_r \\ 0 & t < t_i \text{ and } t < t_r \\ 0 & t \ge t_i \text{ and } t \ge t_r \end{cases} \tag{5.1}$$

The function F_i is the step function over time expected for the i^{th} satellite. The time component is measured in epochs, and is represented by t. The variable t_i is the time in epochs that the i^{th} satellite is crossed by the wall, that is shown by Equation 5.2. The time that the reference clock is crossed by the wall is t_r. The variable h is the height of the signal bias between the reference clock and the i^{th} satellite, which for simplicity is set to one.

$$t_i(v_x, t_r, r_i, r_{ref}) = t_r + \frac{r_i - r_{ref}}{v_x} \tag{5.2}$$

The time the i^{th} satellite is crossed by the wall can be measure from t_r if the exact positions along the direction of the wall velocity are known (r_i, r_{ref}), and the magnitude of the velocity of the wall is known (v_x). This reduces the only unknown time parameter to t_r. The positions (r_i, r_{ref}) are given from the dot product of the ECI position vector and the direction of wall propagation, which is a function of the two angular parameters (θ, ϕ) in Equation 5.3.

$$r_{i,ref}(\theta, \phi) = \begin{pmatrix} x_{i,ref} \\ y_{i,ref} \\ z_{i,ref} \end{pmatrix} \cdot \begin{pmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{pmatrix} \tag{5.3}$$

The position vectors of all the satellites are given from JPL's GPS data files, which are open to public access. The problem is now reduced to four unknown parameters (v_x, t_r, θ, ϕ).

Clock bias data generally has a random walk component, and thus is non-stationary, which poses problems for the application of statistical methods. It is convenient in this case to look at the differenced signal between neighboring epochs, which can be thought of as taking a discrete derivative. After differencing, the signal only has values at the epochs where the satellites were affected by the wall. This makes the signal look simpler, makes it easier to analyze, and makes the noise in the data approximately white. Such a signal is plotted in Figure 5.4 that has a functional form from Equation 5.4, using i and j to denote the i^{th} satellite and the j^{th} epoch.

$$s_{ij} = F_i(j, t_i, t_r, h) - F_i(j-1, t_i, t_r, h) \tag{5.4}$$

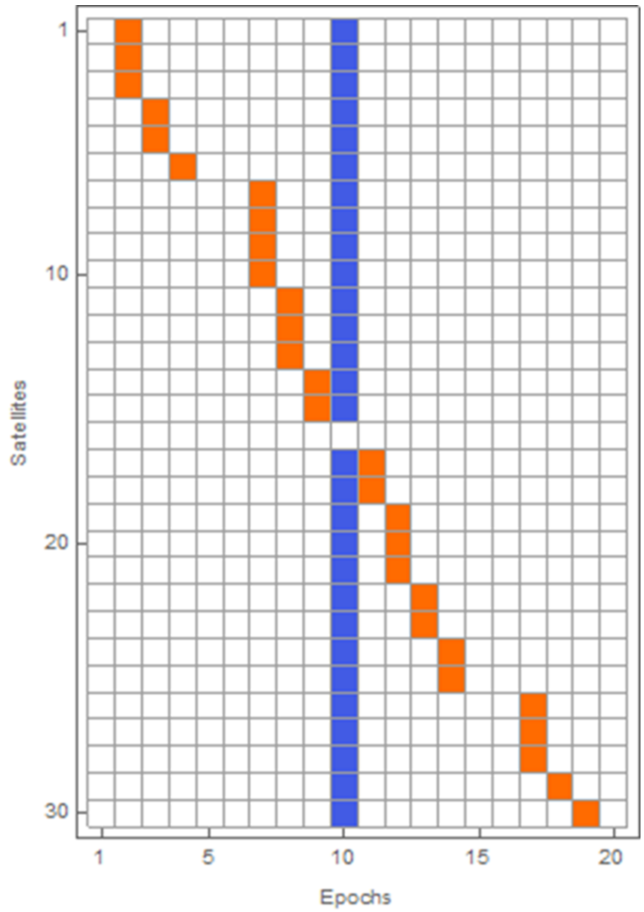


Figure 5.5: *Differenced signal corresponding to Figure 5.3.*

5.2 Unknown Thin Wall Parameters

Each signal will look different depending on the unknown parameters (v_x, t_r, θ, ϕ) involved with the wall. The first parameter is the magnitude of the velocity of the wall with respect to the Earth. The wall can be oriented in any way in space and the velocity vector of the wall can be pointing at an angle with respect to the normal vector of the wall as shown in Figure 5.5. From our perspective, however, a randomly oriented wall with a random velocity vector will look the same as wall traveling with a velocity perpendicular to the wall, so we are only sensitive to the perpendicular component of the velocity vector. From the Standard Halo Model of DM in our galaxy, one can infer the probability distribution of velocities one would expect from walls passing through our solar system. The range of the magnitude of these velocities is approximately 10-700 km/s with respect to the Earth, as seen from Figure 5.6.

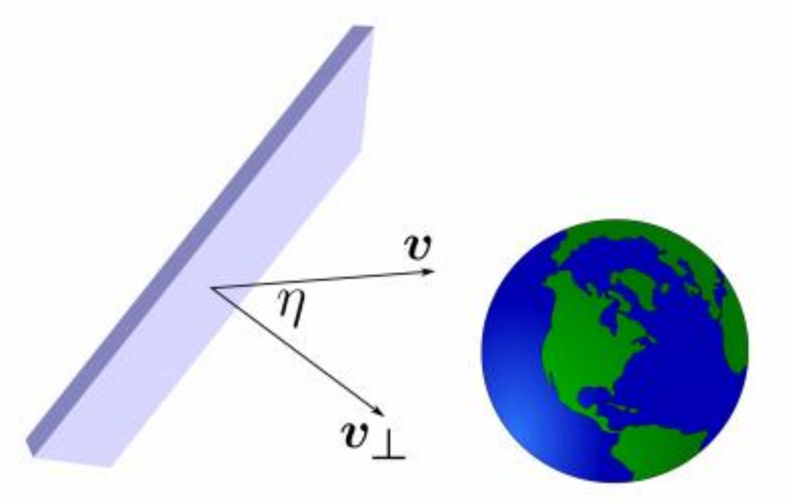


Figure 5.6: *Example of a thin TD wall crossing the Earth.*

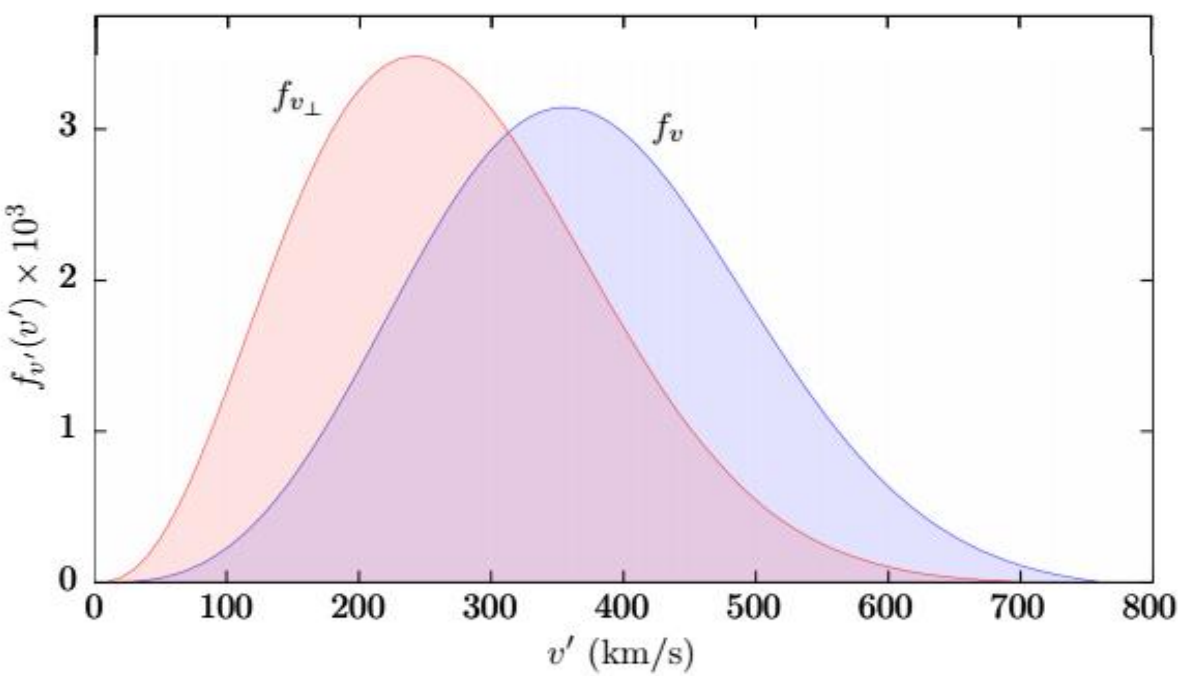


Figure 5.7: *Probability density function of velocities and normal velocities of thin walls [10].*

The next parameter that will affect the signal is the exact time the reference clock is crossed by the wall, denoted as t_r. If this variable is changed slightly, other jump measurements might be picked up in different epoch slots, which will change the signal depending on the exact times the rest of the satellites were affected. GPS data is collected and reported accumulatively, so that all the data collected in the thirty seconds before the epoch ends is reported in that epoch slot.

Next, although the solar system barycentric vector is the most likely direction of a DM event on average, one could theoretically come from any direction. To be completely sure that we can cover every possible signal, we must consider two more variables, the two angles in spherical coordinates (θ, ϕ) that can change the direction the wall comes from. θ is defined as the polar angle, that is measured from the ECI z-axis, and ϕ is defined as the equatorial angle, that is measured from the ECI x-axis along the equatorial plane. In effect, this only changes the order that the DM wall affects the satellites, but we must still take the whole sphere into account.

5.3 Characteristics of the DM Search Function

The time bias data streams that we intend to search through can have any value at the i^{th} satellite and the j^{th}epoch, but our generated signals can only have values of 1, -1, or 0. Before we search through these data for a signal, we must first convert these data (d_{ij}) to a trinary form (d_{ij}^{tri}) using Equation 5.5, where the cutoff magnitude s_{cut} is set to one standard deviation of the data, which is typically 0.1-ns, as it will be difficult to resolve events if they are this close to the noise level.

$$d_{ij}^{\text{tri}} = \begin{cases} 1 & d_{ij} \geq s_{cut} \\ -1 & d_{ij} \leq -s_{cut} \\ 0 & -s_{cut} < d_{ij} < s_{cut} \end{cases} \tag{5.5}$$

A complete Bayesian analysis would require a quadruple integral over the four unknown parameters to find an event, and be confident that it is indeed an event.

$$\int_0^{2\pi} \int_0^{\pi} \int_{50}^{51} \int_{10}^{700} dv_x dt_r d\theta d\phi \tag{5.6}$$

Instead of doing these computationally intensive integrals however, it is more practical to find a search method that is fast and effective for finding potential events, while this Bayesian method is in production. To find the best fitting signal in some data stream, we need a function that will give a large value for a well-fitting signal, and a small one for a poorly-fitting signal. Such a function should be designed such that the matrix of data points we have for a set of epochs can be checked against a possible signal, and give a value as to how much of that signal exists in the data. The function I used is derived from a Gaussian probability curve.

$$e^{-\sum_{ij}\left(d_{ij}^{\text{tri}} - s_{ij}(v_x, t_r, \theta, \phi)\right)^2} \tag{5.7}$$

In this example of function, any deviation from a perfect signal makes the exponential sum smaller, so data that is nothing like the signal will be very small in comparison to data that has any indication of a signal. This function is a four-dimensional surface imbedded in a five-dimensional parameter space that is unique for every data stream. One would be interested in where the value of this function is at a global maximum, which would correspond to the four signal parameters that fit the data the best. To find this global maximum, I used a numerical maximization function in Wolfram Mathematica and used a method called Differential Evolution that I found to give the best success in finding a global maximum in test data. This method also works well on non-differentiable functions, which in this case our function will not be differentiable at every point since due to the sum in the exponent, the function can only take certain "quantized" values, so in high resolution and in a small region, it will look "choppy." Differential Evolution will always look for a global maximum, but is not guaranteed to find one [11].

Once a maximum has been found, there needs to be a way to tell how well the signal fits the data, so that we can discard data that doesn't fit the thin wall model. To do this we use the divide by the function value with no signal component.

$$O = \frac{e^{-\sum_{ij}\left(d_{ij}^{\mathrm{tri}} - s_{ij}(v_x,t_r,\theta,\phi)\right)^2}}{e^{-\sum_{ij}\left(d_{ij}^{\mathrm{tri}}\right)^2}} \tag{5.8}$$

I used an extensive amount of testing to make sure this algorithm worked as designed. Wyatt Williams created test data with a data simulator he designed. These test data included files that contained signals using randomly generated parameters, and some that did not, which were each overlaid with random Gaussian noise with a standard deviation σ. Three trials were conducted to search for the events in this test data, one with signals at a height 5σ, one with 3σ, and one with σ, each with thirty files generated. The results were then checked after the search with a key that contained the parameters the files were generated with, that were kept secret throughout the search. To check whether an event had been found correctly or not, I looked at the result of Equation 5.8. If its value is sufficiently large, it is very likely that an event has been found.

We have noticed however, that due to the need for a reference clock to make the bias data, there are frequent vertical lines in the data matrix (shown in Figure 5.7), which corresponds to every satellite jumping within the same epoch. This happens most likely because of some perturbation to the reference clock, which makes the bias look approximately the same for every satellite in that epoch. Since every signal includes a vertical line of this sort (the blue line in Figure 5.4), it has the potential to lead to many false positives, so instead of weighting the odds ratio with signal in the denominator, I weight it with a vertical line at the centered epoch. This eliminates events being found due just to this vertical line.

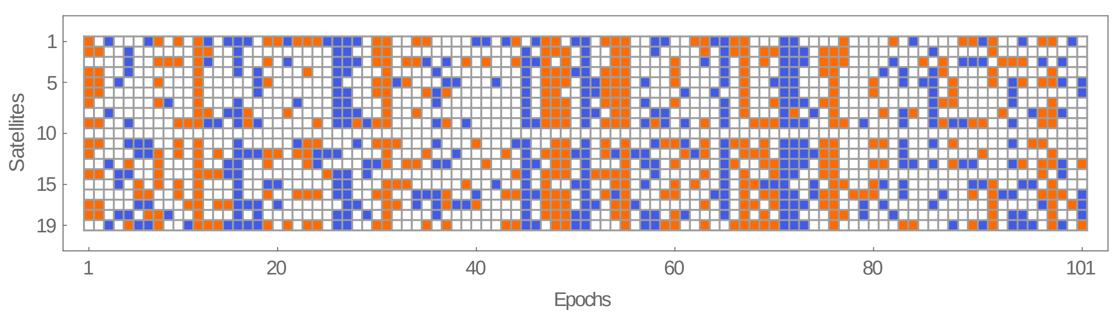


Figure 5.8: *Sample Data of a random day shows frequent vertical lines*

	Correct Positives	Correct Negatives	False Positives	False Negatives
5σ	14	15	0	1
3σ	13	14	0	3
σ	1	15	0	14

Table 5.1: *Results of the simulator test.*

The results of Table 5.1 show that at 5σ, the program has a success of 97%, at 3σ it has 90%, but once the signal gets close to the noise it becomes nearly impossible to find the signal. It is also interesting to note that even under the circumstances of σ events, I never found a false positive, meaning it is somewhat rare to find an event out of randomly generated noise.

It is with this mathematical algorithm that is applied to searched for a signal in each set of data. It takes a lot of computing power to compute this maximum however, so many processor cores are used to speed up the process.

6 Search Results

The results of the search gave 14 candidate events, all given in Appendix C. These events do not fit the thin wall model completely. The found events have a central blue vertical line that corresponds to the reference clock being crossed by the wall, but this line has a much higher magnitude than the orange diagonal line of each satellite being crossed, demonstrated by Figure 6.1. The top plot is the signal that corresponds to the four parameters (v_x, t_r, θ, ϕ) that the program chose as the best fitting values. The middle plot is the actual data from the file being tested. The bottom plot is the matrix that corresponds to the signal fit to the data. The difference between the left and right frames is the s_{cut} magnitude used. The lower cut shows a good fit, but the higher cut shows hardly any signal left, while the vertical line remains. This suggests some perturbation to the reference clock instead of an event.

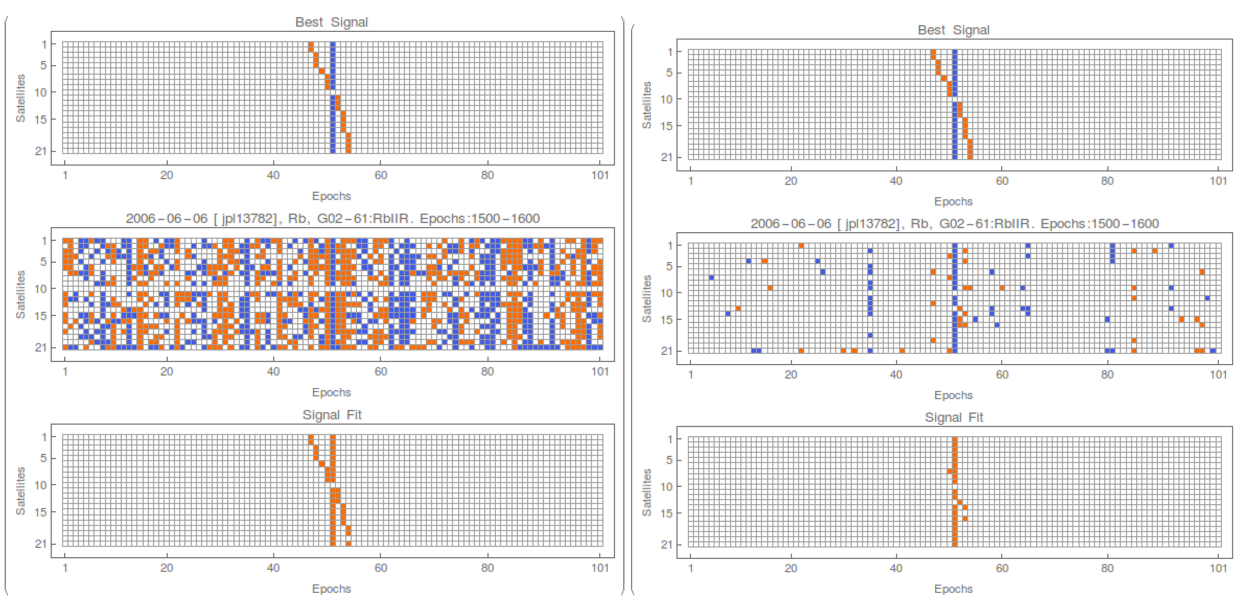


Figure 6.1: *Demonstration of the difference in signal fit at a s_{cut} of 0.1 ns (left) and 0.2 ns (right).*

7 Conclusion

The GPS.DM observatory will remain dedicated to searching GPS atomic clock data for signatures of DM. There remain many different models yet to explore and DM candidate limits to improve upon. My research improved the limits on TD thin wall interactions with atomic clocks and found more than a dozen candidate events. These events do not fit the thin wall model; however, they may fit other models such as monopoles and thick walls. Our future work will be to apply these models to our search and to the found events, to be completely certain that we can either rule these events out, or accept them as statistically fitting a model. The GPS.DM observatory will also focus on a complete Bayesian search in the future to improve upon this work.

References

[1] A. Derevianko and M. Pospelov, "Hunting for topological dark matter with atomic clocks," *Nature Physics,* vol. 10, no. 12, pp. 933-936, 2014.

[2] H. D. Curtis, Orbital Mechanics for Engineering Students, Butterworth-Heinemann, 2005.

[3] "Wikimedia Commons," 24 January 2017. [Online]. Available: https://commons.wikimedia.org/wiki/File:Orbit1.svg. [Accessed 18 April 2017].

[4] R. B. Stothers, "Galactic disk dark matter, terrestrial impact cratering and the law of large numbers," *Mon. Not. Roy. Astron. Soc.,* vol. 300, no. 4, pp. 1098-1104, 1998.

[5] J. Liu, Z. Zhu and H. Zhang, "Reconsidering the Galactic Coordinate System," *Astronomy and Astrophysics,* vol. 526, pp. A16 1-7, 2011.

[6] W. Dehnen and J. Binney, "Local Stellar Kinematics from Hipparcos Data," *Mon. Not. Roy. Astr. Soc.,* vol. 298, no. 2, pp. 387-394, 1998.

[7] "JPL Horizons Web Interface," Jet Propulsion Laboratory , [Online]. Available: https://ssd.jpl.nasa.gov/horizons.cgi. [Accessed 18 4 2017].

[8] "Constellation Guide," 2017. [Online]. Available: http://www.constellation-guide.com/constellation-list/cygnus-constellation/. [Accessed 18 April 2017].

[9] J. Camparo, "The rubidium atomic clock and basic research," *Physics Today,* pp. 33-39, November 2007.

[10] B. Roberts, G. Blewitt, C. Dailey, M. Murphy, M. Pospelov, A. Rollings, J. Sherman, W. Williams and A. Derevianko, "GPS as a dark-matter detector: Orders-of-magnitude improvement on couplings of clumpy dark matter to atomic clocks," *submitted to Nature Communications,* preprint arXiv: hep-ph/1704.06844.

[11] R. Storn and K. Price, "Differential Evolution – A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces," *Journal of Global Optimization,* vol. 11, no. 4, pp. 341-359, 1997.

Appendix A: Orbital Elements Conversion Equations and Mathematica Code

Kepler elements $\longrightarrow$ Cartesian position and velocity

formula sheet **Nico Sneeuw**

Problem: Given 6 Kepler elements $(a, e, I, \omega, \Omega, M)$, find the corresponding inertial position $\boldsymbol{r}$ and velocity $\dot{\boldsymbol{r}}$.

Solution (Kaula,1966): First get the eccentric anomaly E from the mean anomaly M by iteratively solving Kepler's equation:

$$E - e\sin E = M \;\Rightarrow\; E_{i+1} = e\sin E_i + M \text{ , with starting value } E_0 = M \tag{1}$$

Next, get the position and the velocity in the q-frame, which has its z-axis perpendicular to the orbital plane and its x-axis pointing to the perigee:

$$\boldsymbol{q} = \begin{pmatrix} a(\cos E - e) \\ a\sqrt{1-e^2}\sin E \\ 0 \end{pmatrix} \;,\; \dot{\boldsymbol{q}} = \frac{na}{1 - e\cos E}\begin{pmatrix} -\sin E \\ \sqrt{1-e^2}\cos E \\ 0 \end{pmatrix} \tag{2}$$

In case the true anomaly ν is given in the original problem instead of the mean anomaly M, the vectors $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ are obtained by:

$$\boldsymbol{q} = \begin{pmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{pmatrix} \;,\; \dot{\boldsymbol{q}} = \frac{na}{\sqrt{1-e^2}}\begin{pmatrix} -\sin\nu \\ e + \cos\nu \\ 0 \end{pmatrix} \tag{3}$$

with

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} \tag{4}$$

The transformation from inertial frame to the q-frame is performed by the rotation sequence $R_3(\omega)R_1(I)R_3(\Omega)$. So, vice versa, the inertial position and velocity are obtained by the reverse transformations:

$$\begin{aligned} \boldsymbol{r} &= R_3(-\Omega)R_1(-I)R_3(-\omega)\boldsymbol{q} && (5) \\ \dot{\boldsymbol{r}} &= R_3(-\Omega)R_1(-I)R_3(-\omega)\dot{\boldsymbol{q}} && (6) \end{aligned}$$

Cartesian position and velocity $\longrightarrow$ Kepler elements

Problem: Given a satellite's inertial position $\boldsymbol{r}$ and velocity $\dot{\boldsymbol{r}}$, find the corresponding Kepler elements $(a, e, I, \omega, \Omega, M)$.

Solution (Kaula,1966): The angular momentum vector per unit mass is normal to the orbital plane. It defines the inclination I and right ascension of the ascending node Ω:

$$\boldsymbol{h} = \boldsymbol{r} \times \dot{\boldsymbol{r}} \tag{7}$$

$$\tan\Omega = \frac{h_1}{-h_2} \tag{8}$$

$$\tan I = \frac{\sqrt{h_1^2 + h_2^2}}{h_3} \tag{9}$$

Rotate $\boldsymbol{r}$ into the p-frame in the orbital plane now and derive the argument of latitude u:

$$\boldsymbol{p} = R_1(I) R_3(\Omega) \boldsymbol{r} \tag{10}$$

$$\tan u = \tan(\omega + \nu) = \frac{p_2}{p_1} \tag{11}$$

The semi-major axis a comes from the total energy and requires the scalar velocity $v = |\dot{\boldsymbol{r}}|$. The eccentricity e needs the scalar angular momentum $h = |\boldsymbol{h}|$:

$$T - V = \frac{v^2}{2} - \frac{GM}{r} = -\frac{GM}{2a} \tag{12}$$

$$a = \frac{GM\, r}{2GM - rv^2} \tag{13}$$

$$e = \sqrt{1 - \frac{h^2}{GM\, a}} \tag{14}$$

In order to extract the eccentric anomaly E, we need to know the radial velocity first:

$$\dot{r} = \frac{\boldsymbol{r} \cdot \dot{\boldsymbol{r}}}{r} \tag{15}$$

$$\cos E = \frac{a - r}{ae} \tag{16}$$

$$\sin E = \frac{r\dot{r}}{e\sqrt{GM\, a}} \tag{17}$$

The true anomaly is obtained from the eccentric one:

$$\tan\nu = \frac{\sqrt{1 - e^2}\sin E}{\cos E - e} \tag{18}$$

Subtracting ν from the argument of latitude u yields the argument of perigee ω. Finally, Kepler's equation provides the mean anomaly:

$$E - e\sin E = M \tag{19}$$

Documentation :

KeplerianElements :

Inputs : position vector (three components), velocity vector (three components)
Overview :
Takes in position and velocity at any time in an orbit and converts them into Keplerian Elements. Returns a list with (in order) the semi - major axis, eccentricity, inclination, argument of the ascending node, argument of perigee, and the mean anomaly.

KeplerianOrbit :

Inputs : position vector (three components), velocity vector (three components), time you want in the orbit (measured in seconds from the time the position and velocity were taken)
Overview : Takes a position and velocity, converts them to elements then time steps the orbit by the desired amount and returns a new position and velocity.

```
(*Defines all rotation matrices around all three axes*)
R1[θ_] := RotationMatrix[-θ, {1, 0, 0}]
R2[θ_] := RotationMatrix[-θ, {0, 1, 0}]
R3[θ_] := RotationMatrix[-θ, {0, 0, 1}]

KeplerElements[r_, rdot_] := Block[
  (*Local Variables*)
  {μ, h, Ω, i, p, u, a, e, rdotN, em, cosem, sinem, v, ω, M, T},

  (*Gravitational Parameter for Earth*)
  μ = 3.98600441 * 10^14;

  (*Finds angular momentum vector per unit mass (normal to orbital plane)*)
  h = r × rdot;

  (*The angular momentum vector defines the inclination i and right
   ascension of the ascending node Ω*)
  Ω = ArcTan[-h[[2]], h[[1]]];
  i = ArcTan[h[[3]], √(h[[1]]^2 + h[[2]]^2)];

  (*Rotates r to the p-frame to find argument of latitude*)
  p = R1[i].R3[Ω].r;
  u = ArcTan[p[[1]], p[[2]]];
```

```
(*Finds semi-major axis from total energy and eccentricity from
  angular momentum*)
a = μ Norm[r] / (2 μ - Norm[r] Norm[rdot]^2);
e = Sqrt[1 - Norm[h]^2 / (μ a)];
(*Finds Radial Velocity*)
rdotN = r.rdot / Norm[r];

(*Finds eccentric anomaly in sine and cosine form to preserve quadrant*)
cosem = (a - Norm[r]) / (a e);
sinem = Norm[r] rdotN / (e Sqrt[μ a]);

(*True anomaly, eccentric anomaly*)
v = ArcTan[cosem - e, Sqrt[1 - e^2] sinem];
em = ArcTan[cosem, sinem];

(*argument of perigee, mean anomaly*)
ω = Mod[u - v, 2 Pi];
M = em - (e sinem);

(*Returns all elements*)
{a, e, i, ω, Ω, M}
]

KeplerianOrbit[r_, rdot_, time_] := Module[
  (*Local Variables*)
  {μ, Ω, i, u, a, em, e, M, newM, ω, n, newem, q, qdot, newr, newrdot},

  (*Generates Keplerian Elements from given initial position and velocity*)
  {a, e, i, ω, Ω, M} = KeplerElements[r, rdot];

  (*Gravitational Parameter for Earth*)
  μ = 3.98600441 * 10^14;
```

```
(*Mean Motion from Kepler's Law*)
n = Sqrt[μ/a^3];

(*Time steps the mean anomaly using mean motion*)
newM = M + n time;

(*Interatively solves Kepler's Equation for the new eccentric anomaly*)
f[e_, em_, M_] := M + e Sin[em];
newem = Nest[Function[em, f[e, em, newM]], newM, 10];

(*Finds the position and velocity in the "q-frame" where the xy
  plane is the orbit plane and the x-axis points at perigee*)
```

$$q = \begin{pmatrix} a\,(\mathrm{Cos}[newem] - e) \\ a\left(\sqrt{(1 - e^2)}\right)\mathrm{Sin}[newem] \\ 0 \end{pmatrix};$$

$$qdot = \left(\frac{n\,a}{1 - e\,\mathrm{Cos}[newem]}\right) * \begin{pmatrix} -\mathrm{Sin}[newem] \\ \left(\sqrt{(1 - e^2)}\right)\mathrm{Cos}[newem] \\ 0 \end{pmatrix};$$

```
(*Rotates back to ECI frame*)
newr = R3[-Ω].R1[-i].R3[-ω].qᵀ[[1]];
newrdot = R3[-Ω].R1[-i].R3[-ω].qdotᵀ[[1]];

(*Returns new position and velocity*)
{newr, newrdot}
]
```

Appendix B: Mathematica Code for DM Thin Wall Signal Search

```
Documentation :

SearchFile :
Inputs : Filename of the data file output from Ben Roberts c++
   code output (must be same syntax)
Overview : Takes the file and generates all of the signals
   possible given the arrangement of satellites and finds the
   best fitting signal,
with an oddsratio. The main program afterward can run multiple
  files at once,
with instructions in comments. You must set the parallel kernels
   settings you want in Edit →
  Preferences → Parallel. Set however many cores your CPU has.
(*Set working directory to the notebook directory*)
SetDirectory@NotebookDirectory[];

(*Set jump height*)
h = 1.;

(*Compiled Equations help the overall time of evaluation*)
(*Equation 5.1*)
F = Compile[
   {t, ti, tr, h},
   ⎡ (h)      ti ≤ t < tr
   ⎢ (-h)     ti > t ≥ tr
   ⎢  0    t < tr && t < ti
   ⎣  0    t ≥ ti && t ≥ tr
  ];

(*Equation 5.2*)
ti = Compile[
   {ri, vx , tr, rref},
   tr + (ri - rref)/vx
  ];
```

```
(*Equation 5.4*)
differencedF = Compile[
   {x, ti, tr, h},
   F[x, ti, tr, h] - F[x - 1, ti, tr, h]
  ];

(*Creates signal point for the ith satellite and jth epoch*)
signalModel = Compile[
   {vx, tr, θ, ϕ,
    {pos, _Real, 2},
    {rrefpos, _Real, 1},
    satelliteNumber,
    epochNumber, {i, _Integer},
    {j, _Integer}},

   Block[
    {n, rproj, rrefproj},

    (*Equation 5.3*)
    n = {Sin[θ] Cos[ϕ], Sin[θ] Sin[ϕ], Cos[θ]} // N;
    rproj = Table[pos[[k]].n, {k, 1, satelliteNumber}];
    rrefproj = rrefpos.n;

    SetPrecision[differencedF[j,
      ti[rproj[[i]], -30 * vx, tr, rrefproj], tr, h], 1]
   ],
   "RuntimeOptions" → {"EvaluateSymbolically" → False}
  ];

(*Makes a signal with only a vertical line in the data
 to weigh with odds ratio*)
nosignalModel[neg_, iref_, satelliteNumber_, epochNumber_,
   jr_, i_, j_] := If[j == jr && i ≠ iref, -neg, 0];

(*Main
 Function: Finds global maximum and oddsratio for a given
   datastream*)
```

```
SearchFile[filename_] := Block[
  (*Local Variables*)
  {oddsRatio, iref, rrefpos, sampleData, satelliteNumber,
   epochNumber,
   pos, gaussPosScore, gaussNegScore, gaussScore, dataFile, Title},

  (*Imports file that contains data*)
  dataFile = Import[filename, "Table"];
  (*Finds number of epochs and satellites in the data*)
  {satelliteNumber, epochNumber} = dataFile[[3, {1, 3}]];
  epochNumber = 101;

  cutoff = 0.1; (*sets data trinary cutoff*)

  jr = 51.; (*sets reference epoch
   (every file has reference epoch in the middle)*)

  vmin = 20.; (*sets bounds for velocity of a wall*)
  vmax = 700.;

  tmin = jr - 1.; (*sets bounds for time the reference clock
   is hit by the wall*)
  tmax = jr;

  θmin = 0.;
  (*sets bounds for the polar component of the direction*)
  θmax = N[Pi];

  ϕmin = 0.;
  (*sets bounds for the longitudinal component of the
   direction*)
  ϕmax = N[2 Pi];

  Title = dataFile[[1, 1]];
  (*Title of the file includes the day and time of the data*)
```

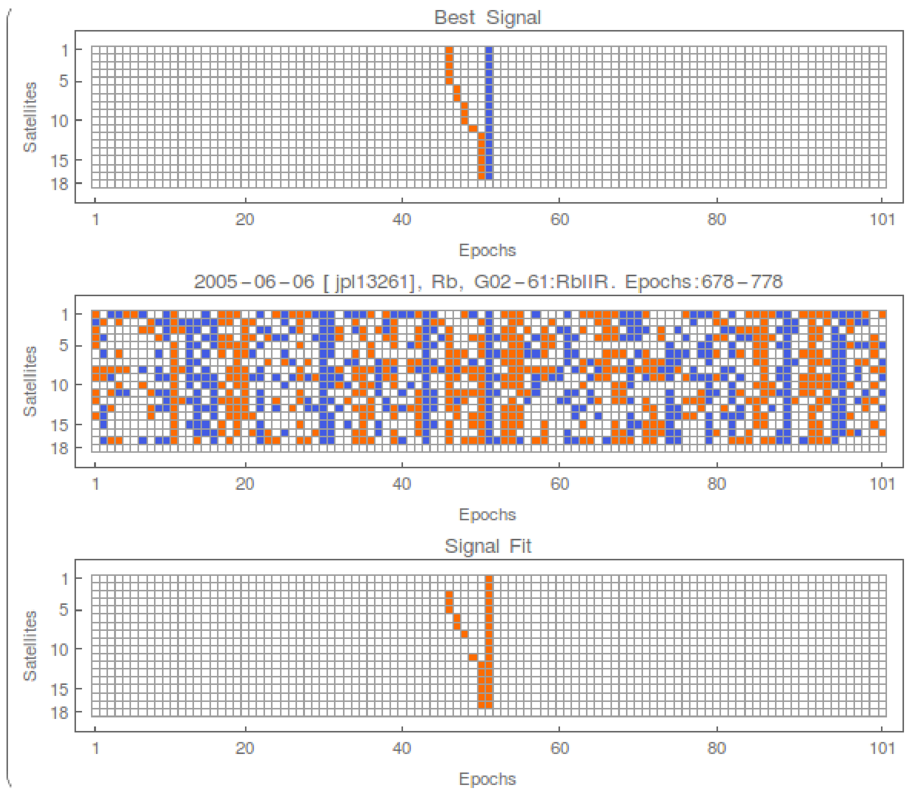
```
pos = dataFile[[4 ;;, 2 ;; 4]];
(*Positions of the satellites in three dimensions*)

(*Finds which of the positions cooresponds to the
 reference clock, seems more complicated than it needs to be*)
rrefName =
 StringTake[
  StringCases[ToString[Title], "Rb, " ~~ x___ ~~ ":Rb" :> x][[1]],
  3];
For[i = 4, i < Length[dataFile], i++,
 If[StringMatchQ[ToString[dataFile[[i, 6]]],
    ___ ~~ rrefName ~~ ___],
  iref = i - 3; rrefpos = dataFile[[i, 2 ;; 4]]
 ]
];

(*Finds time series data and makes it trinary (Equation 5.5)*)
sampleData = dataFile[[4 ;;, 7 ;;]];
sampleData = Table[
  If[sampleData[[i, j]] ≥ cutoff,
   1.,
   If[sampleData[[i, j]] ≤ -cutoff,
    -1.,
    0.]
  ],
  {i, 1, satelliteNumber} , {j, 1, epochNumber}
 ];

(*Sets test function (Equation 5.7)*)
gaussFunction[vx_, tr_, θ_, ϕ_, neg_] :=
 Exp[
  -Sum[
    (sampleData[[i, j]] -
       neg * signalModel[vx, tr, θ, ϕ, pos, rrefpos,
         satelliteNumber, epochNumber, i, j])^2,
    {i, 1, satelliteNumber} , {j, 1, epochNumber}
```

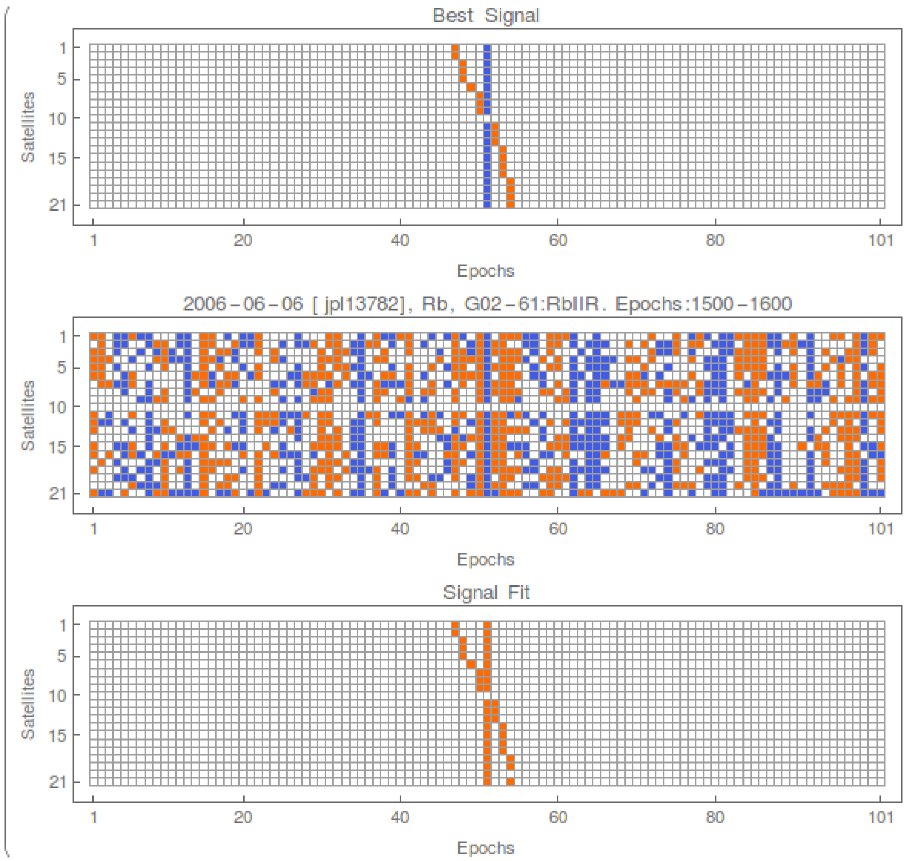
```
    ]
  ];

(*sets parallel kernel divisor*)
KernelNumber = $ConfiguredKernels[[1, 1]];

(*Divides the parameter space into sections that are
 tested by each parallel kernel*)
dv = (vmax - vmin)/KernelNumber;
vTable = Table[{i, i + dv}, {i, vmin, vmax - dv, dv}];

(*Finds global maximum of the test function in parallel*)
MaxValuesTable = ParallelTable[
  NMaximize[{gaussFunction[vx, tr, θ, ϕ, neg],
    vTable[[i, 1]] ≤ vx ≤ vTable[[i, 2]], tmin ≤ tr ≤ tmax,
    θmin ≤ θ ≤ θmax, ϕmin ≤ ϕ ≤ ϕmax, neg == 1. || neg == -1.},
   {vx, tr, θ, ϕ, neg}, Method -> "DifferentialEvolution"]
  , {i, 1, KernelNumber}
 ];

(*MaxValuesTable has a bunch of maximums from each kernel,
this finds the largest one*)
BestFitMaximum = MaximalBy[MaxValuesTable, First][[1]];
(*sets the negative element since we know what it is*)
tempneg = BestFitMaximum[[2, 5]][[2]];
(*sets denominator of oddsratio*)
gaussNegScore = Exp[
  -Sum[
    (sampleData[[i, j]] - nosignalModel[tempneg, iref,
        satelliteNumber, epochNumber, jr, i, j])^2,
    {i, 1, satelliteNumber} , {j, 1, epochNumber}
   ]
  ];
(*oddsratio*)
oddsRatio = BestFitMaximum[[1]]/gaussNegScore;
```

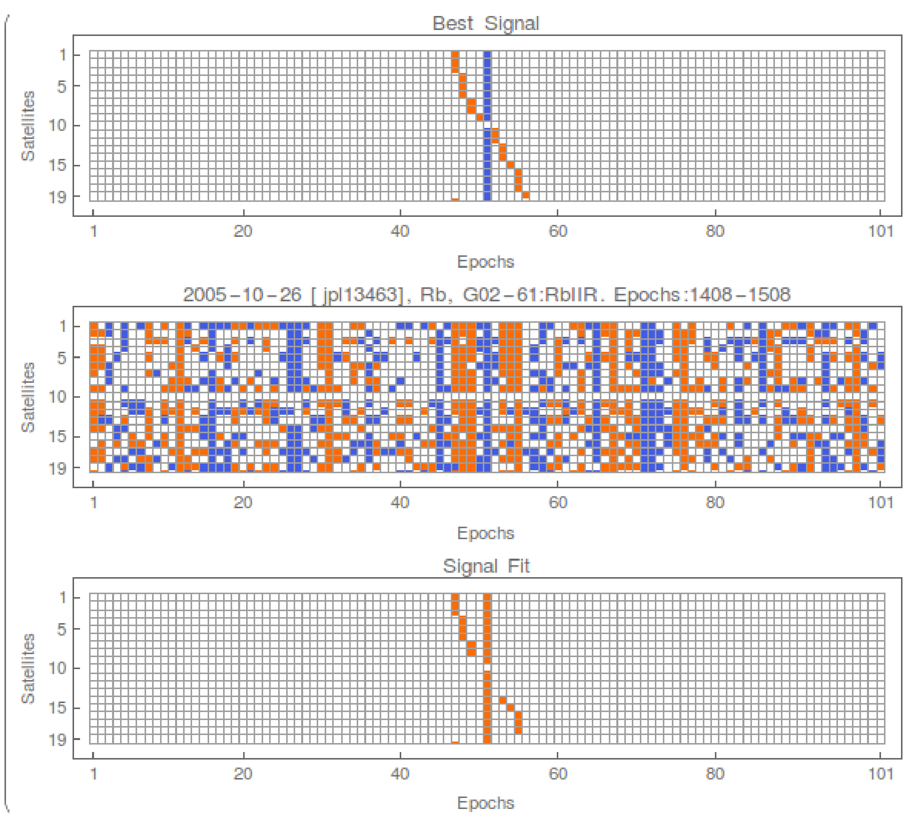
```
  (*Returns likelihood and best maximum*)
  {oddsRatio, BestFitMaximum}

 ]
(*Main
 Program: loops through all data files and applies SearchFile
   to each one, when a progress indicator and Estimated
 time of completion*)
ta = AbsoluteTime[];
outputName = ""; (*Insert output filename tag*)
ResultsDirectory = "";
(*Create and insert a directory to put the results in*)
DataDirectory = "";
(*Insert directory where the data files are*)
inputlist = FileNames["*", DataDirectory];
numFiles = Length[inputlist];
iFiles = 1;
Dynamic[ProgressIndicator[iFiles, {1, numFiles}]]
(*Progress indicator based on files completed*)
(*Main Program Loop*)
For[iFiles = 1, iFiles ≤ numFiles, iFiles++,
 tc = AbsoluteTime[];
 output = SearchFile[inputlist[[iFiles]]];
 td = AbsoluteTime[];
 title =
  StringCases[inputlist[[iFiles]],
    "/pat" ~~ x__ ~~ ".txt" :> x][[1]];
 NotebookFind[SelectedNotebook[], "Print", All, CellStyle];
 NotebookDelete[];
 Print["Estimated Completion Time: " <>
   DateString[AbsoluteTime[] + (td - tc) * (numFiles - (iFiles))]];
 Export[ResultsDirectory <> outputName <> "-" <> title <> ".txt",
  output] (*Makes a new file for each result*)
]
tb = AbsoluteTime[];
t = tb - ta (*Outputs exact time it ran for in seconds*)
```

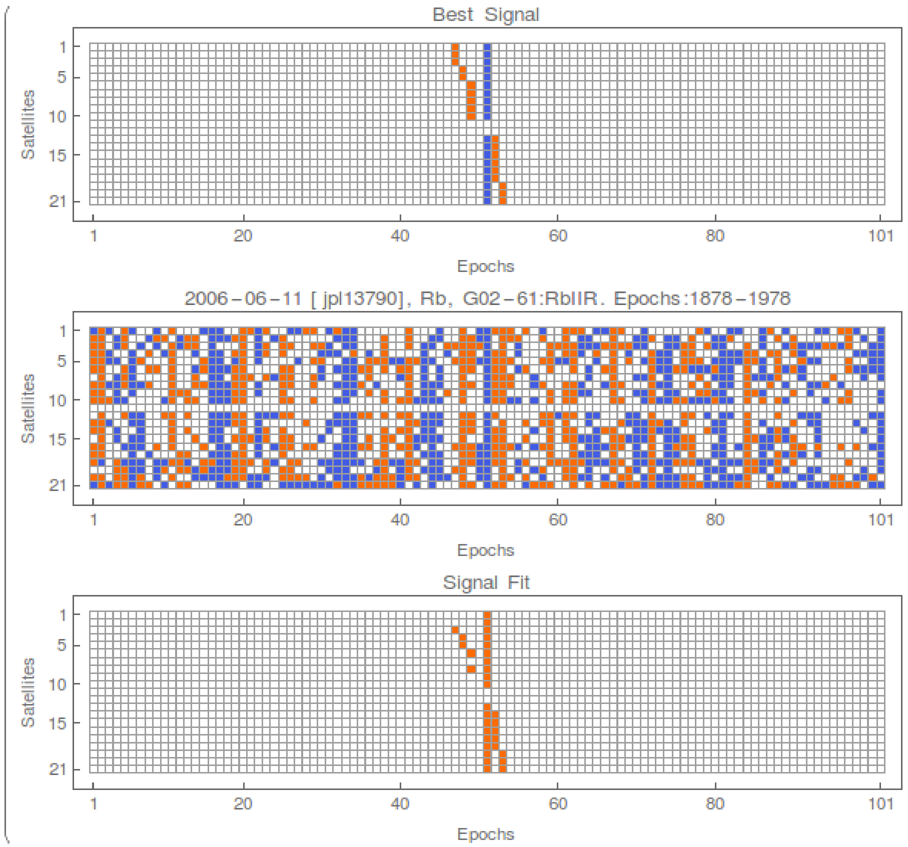
Appendix C: Search Results with the Highest Matching Signals

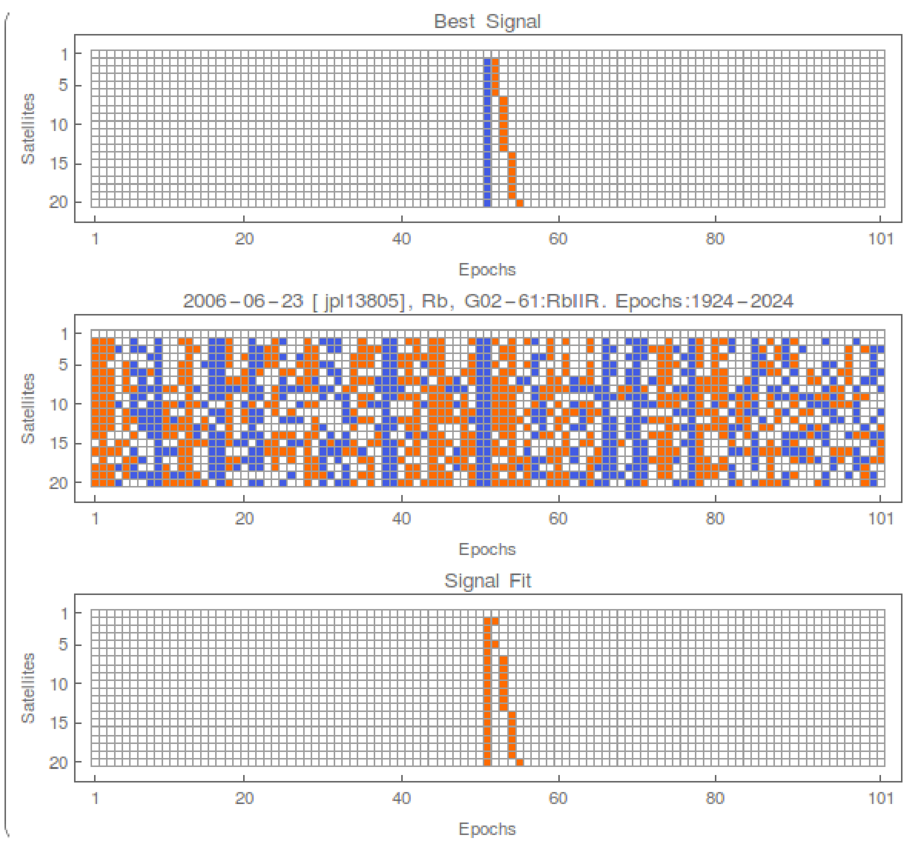
This appendix contains the best fitting signals found by the search program in Appendix B. Each event shown contains three matrix plots. The first plot is the signal that corresponds to the four parameters (v_x, t_r, θ, ϕ) that the program chose as the best fitting values. The second plot is the actual data from the file being tested, made trinary with Equation 5.5. The third plot is the matrix that corresponds to every element of the previous two matrices being multiplied together, which effectively shows the how much of the signal matches the data. The middle plot contains a title at the top of it that shows the date and epoch numbers of the data that the event was found in, along with the satellite that was chosen as the reference clock.

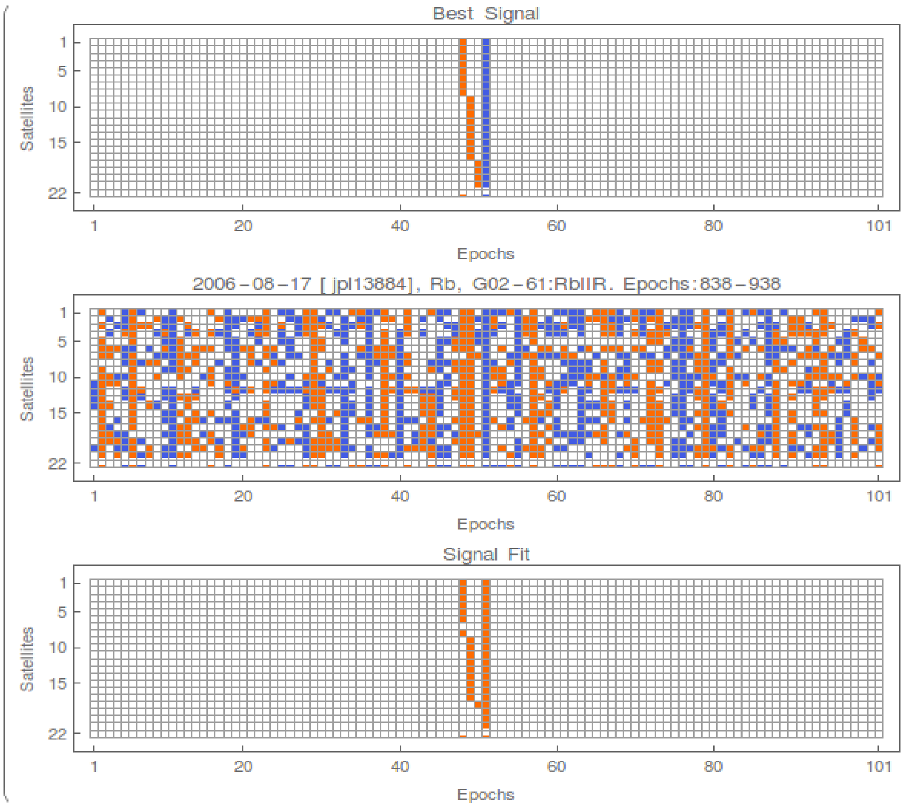


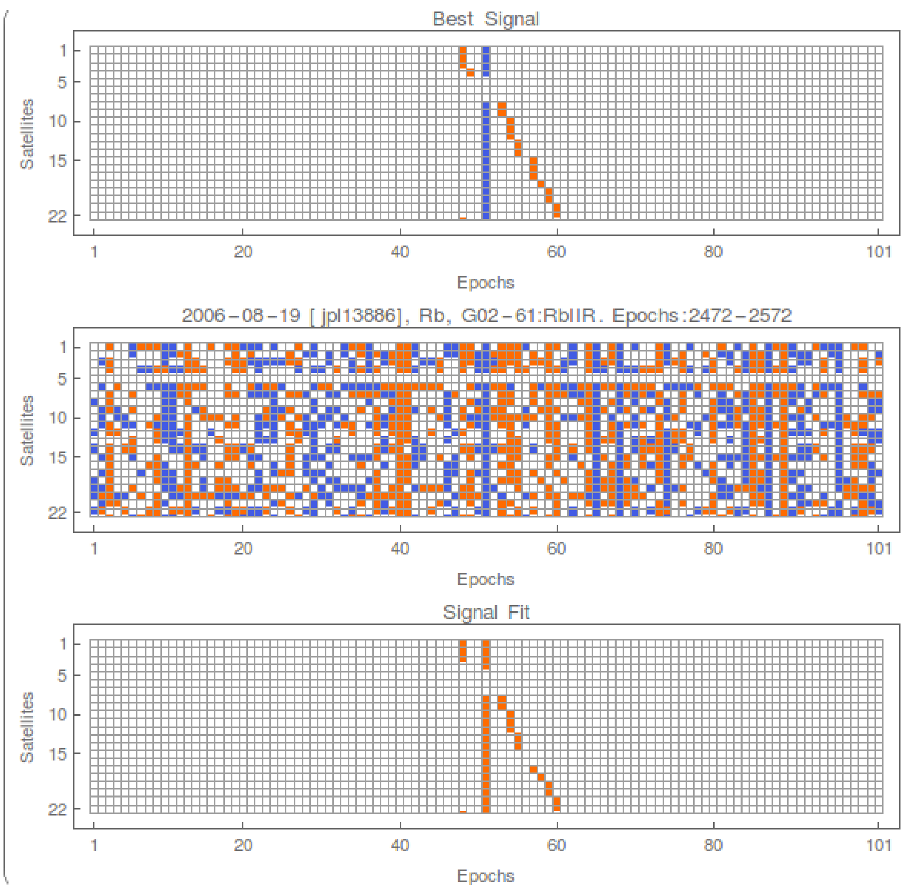

Best Signal
Satellites
Epochs
2006–06–06 [jpl13782], Rb, G02–61:RbIIR. Epochs:1500–1600
Satellites
Epochs
Signal Fit
Satellites
Epochs

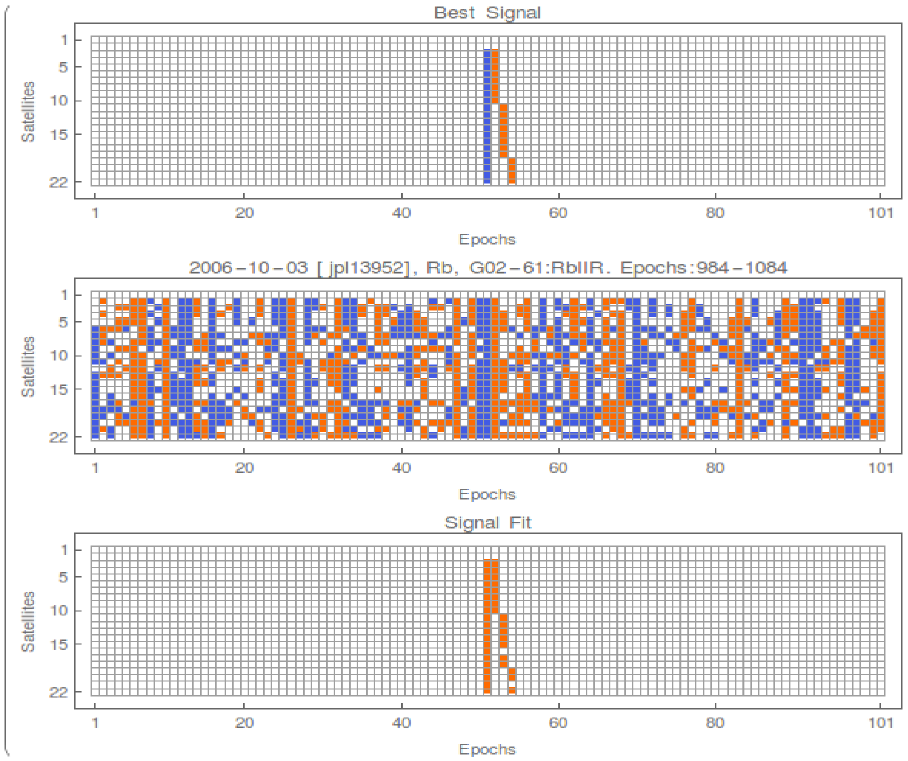

Best Signal
Satellites
Epochs
2005–10–26 [jpl13463], Rb, G02–61:RbIIR. Epochs:1408–1508
Satellites
Epochs
Signal Fit
Satellites
Epochs

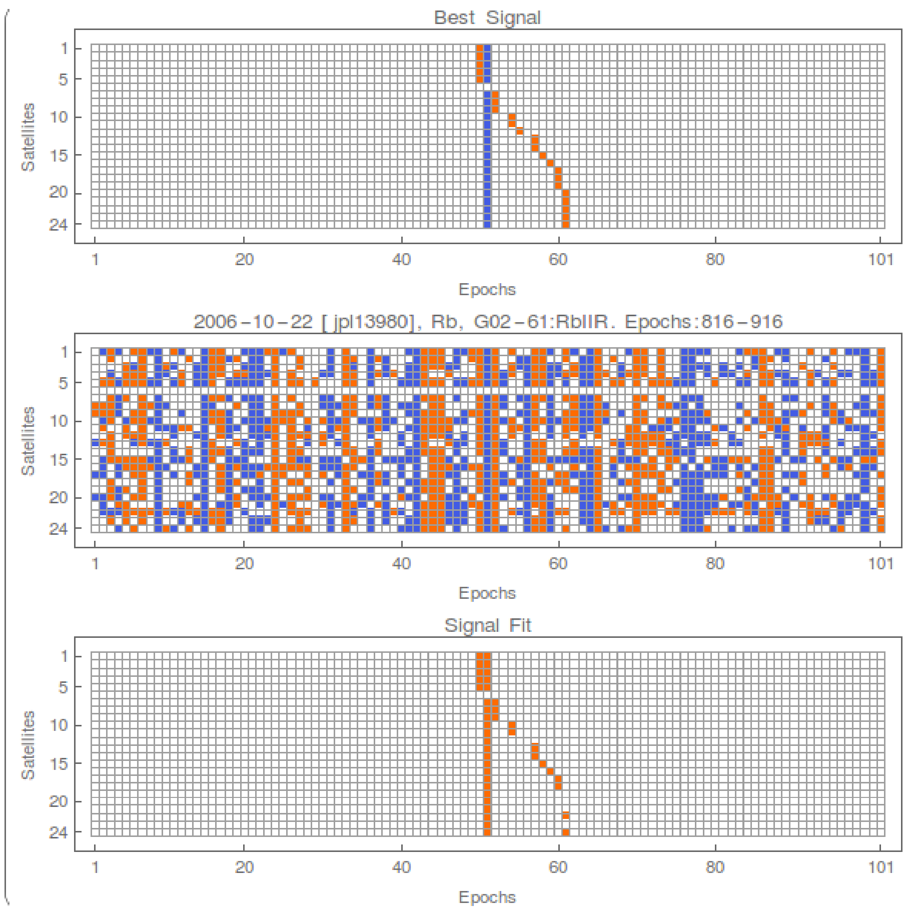

Best Signal
Satellites
Epochs
2006-06-11 [jpl13790], Rb, G02-61:RbIIR. Epochs:1878-1978
Satellites
Epochs
Signal Fit
Satellites
Epochs

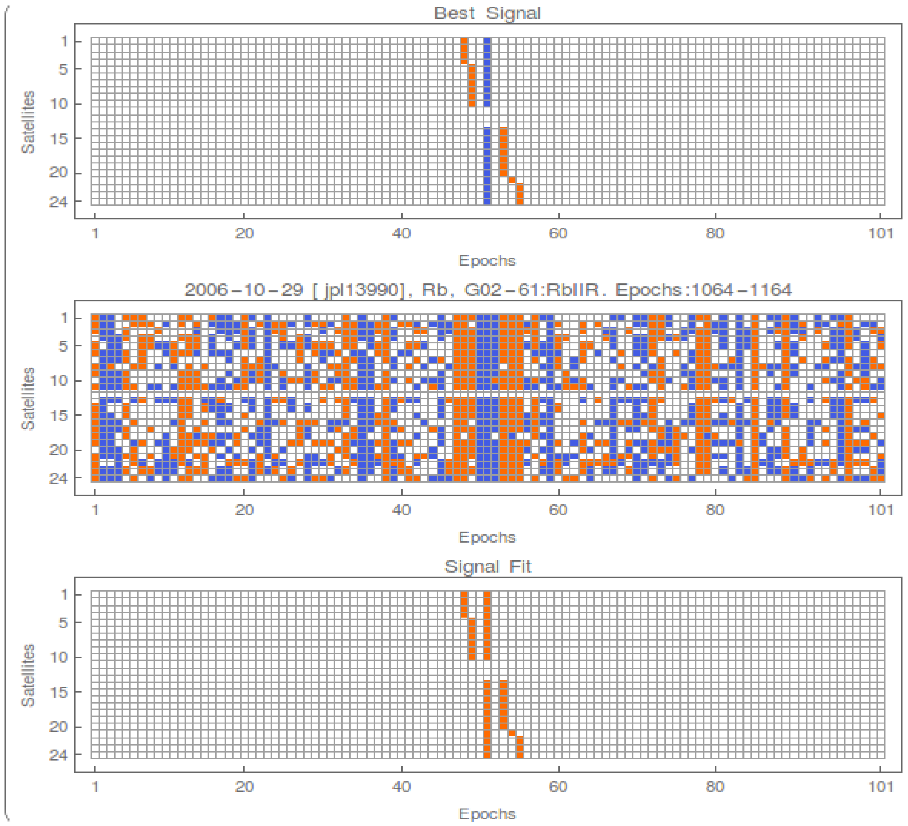

Best Signal
Satellites
Epochs
2006-06-23 [jpl13805], Rb, G02-61:RbIIR. Epochs:1924-2024
Satellites
Epochs
Signal Fit
Satellites
Epochs

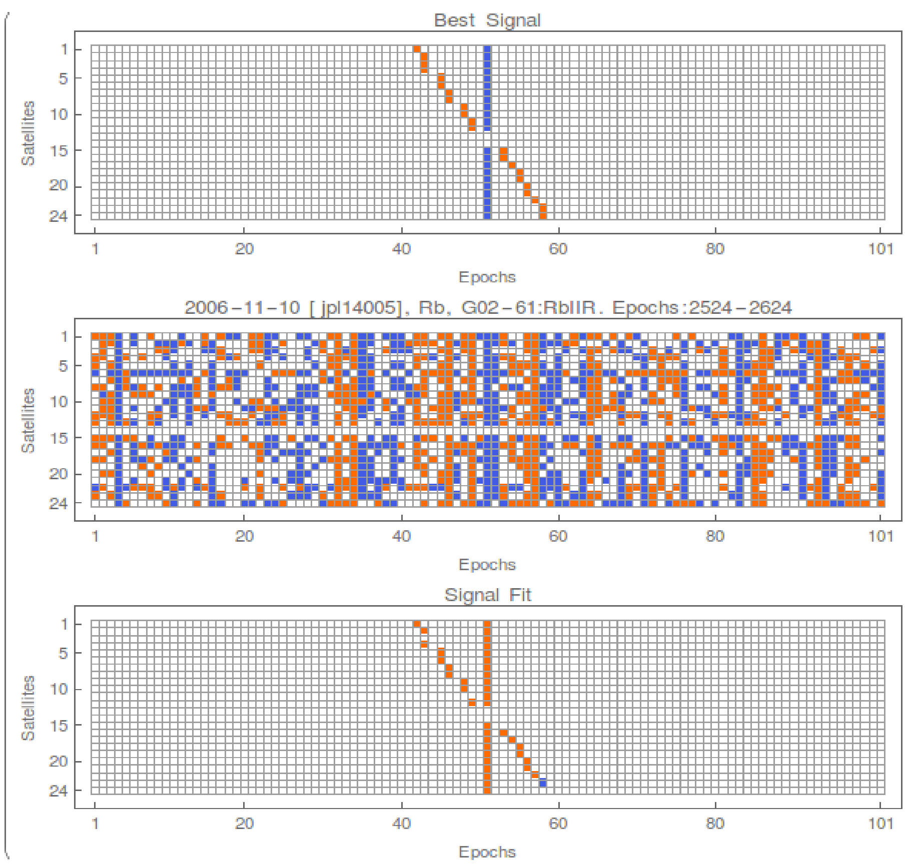

Best Signal
Satellites
1
5
10
15
22
1
20
40
60
80
101
Epochs
2006-08-17 [jpl13884], Rb, G02-61:RbIIR. Epochs:838-938
Signal Fit

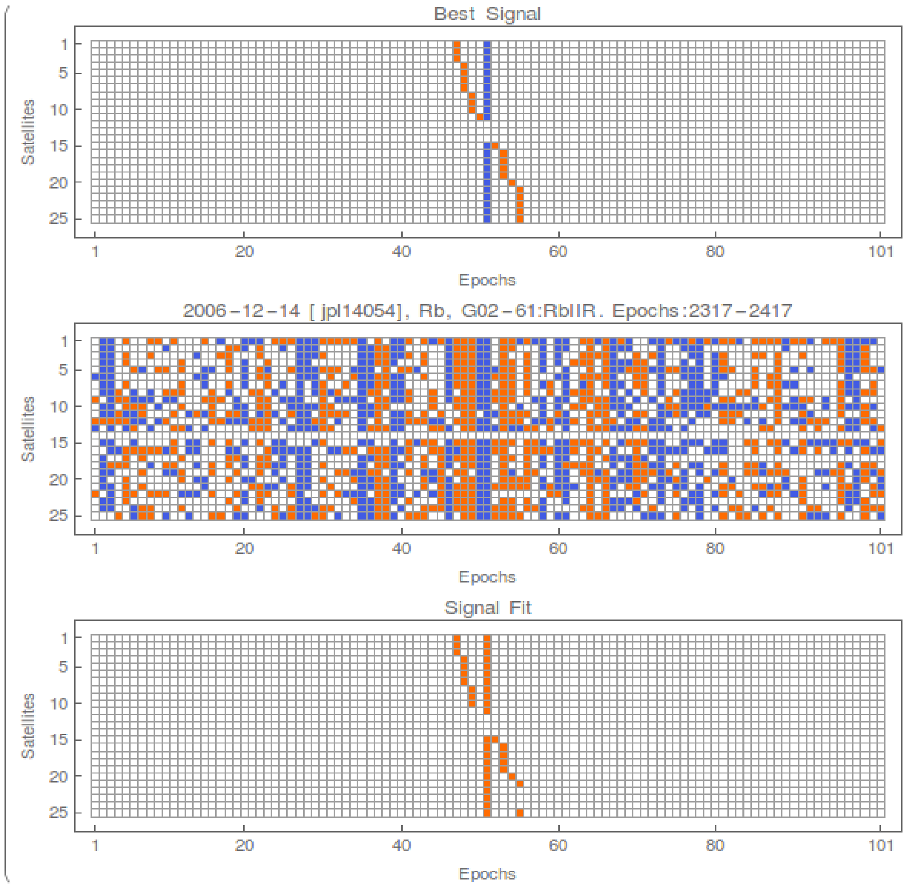

Best Signal
Satellites
1
5
10
15
22
1
20
40
60
80
101
Epochs
2006-08-19 [jpl13886], Rb, G02-61:RbIIR. Epochs:2472-2572
Signal Fit

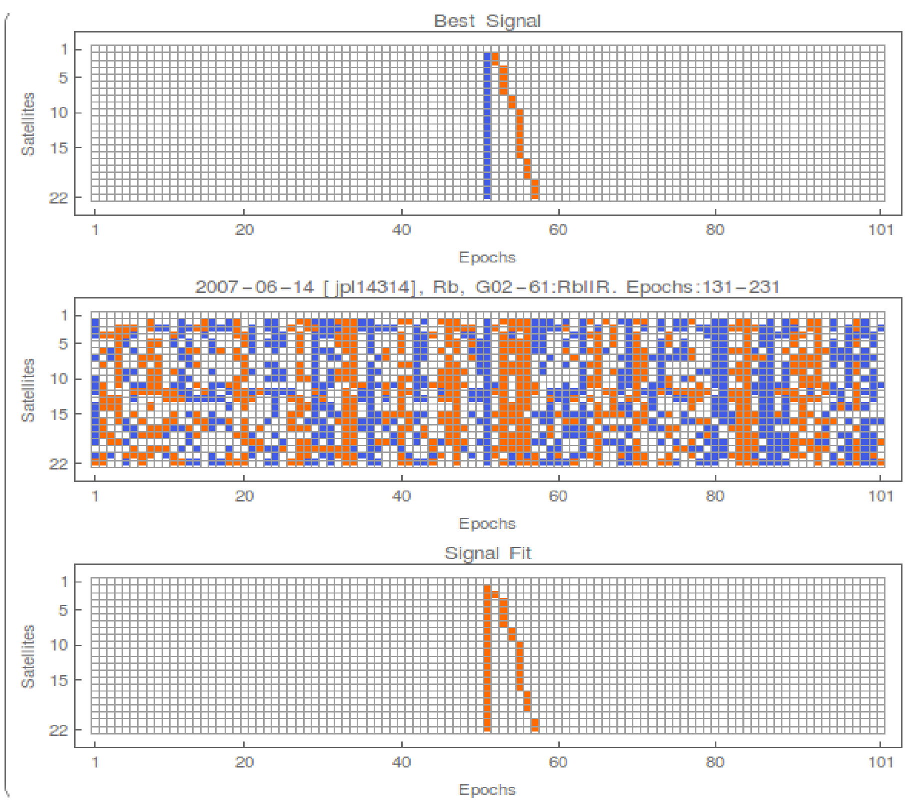

Best Signal
Satellites
Epochs
2006-10-03 [jpl13952], Rb, G02-61:RbIIR. Epochs:984-1084
Satellites
Epochs
Signal Fit
Satellites
Epochs

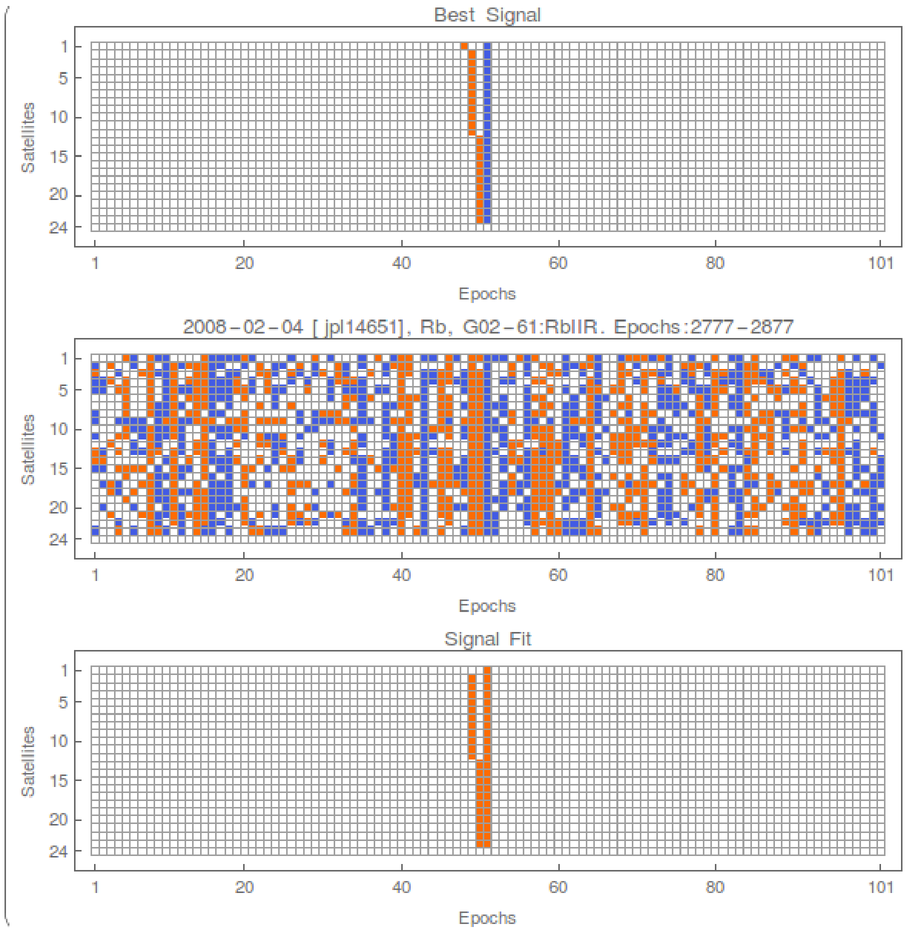

Best Signal
Satellites
Epochs
2006-10-22 [jpl13980], Rb, G02-61:RbIIR. Epochs:816-916
Satellites
Epochs
Signal Fit
Satellites
Epochs


Best Signal
Satellites
Epochs
2006-10-29 [jpl13990], Rb, G02-61:RbIIR. Epochs:1064-1164
Satellites
Epochs
Signal Fit
Satellites
Epochs


Best Signal
Satellites
Epochs
2006-11-10 [jpl14005], Rb, G02-61:RbIIR. Epochs:2524-2624
Satellites
Epochs
Signal Fit
Satellites
Epochs


Best Signal
Satellites
Epochs
2006 – 12 – 14 [jpl14054], Rb, G02 – 61:RbIIR. Epochs:2317 – 2417
Satellites
Epochs
Signal Fit
Satellites
Epochs


Best Signal
Satellites
Epochs
2007 – 06 – 14 [jpl14314], Rb, G02 – 61:RbIIR. Epochs:131 – 231
Satellites
Epochs
Signal Fit
Satellites
Epochs



Best Signal
Satellites
Epochs
2008-02-04 [jpl14651], Rb, G02-61:RbIIR. Epochs:2777-2877
Satellites
Epochs
Signal Fit
Satellites
Epochs