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A GENERALIZED THEORY OF STARK BROADENING OF HYDROGEN-LIKE SPECTRAL LINES IN DENSE PLASMAS

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Abstract—An improved semiclassical theory of Stark broadening of spectral lines emitted by hydrogen-like ions is developed in spirit of our previous papers. Compared to the standard theories, the improvement is achieved by taking into account on equal footing a "dynamic" splitting of Stark sublevels caused by one of the components of the electron microfield and a quasistatic splitting of Stark sublevels caused by ion microfield (only the latter was allowed for in the standard theories). The presented generalized theory is developed analytically to the same level as the standard theories and embraces the latter as one of its limiting cases corresponding to relatively low densities of a plasma. However, for dense plasmas the predictions differ: e.g., for lines with intense central Stark components (such as L_α, L_γ, H_α, etc.) the standard theories noticeably underestimate Stark broadening for high density plasmas. For conditions of the experiment by Grützmacher and Johannsen previous calculations ended up with halfwidths of the H_α line of HeII that were drastically smaller (by factor of two) than the observed halfwidth of this line. Calculations by our generalized theory result in a better agreement with the experiments.

1. INTRODUCTION

In our previous papers[1–3] we had developed a series of generalized impact broadening theories that are more accurate than the Standard Semiclassical Theories (SST) of impact broadening.[4–6] From the physical point of view, the enhanced precision was achieved using the lowering of the spherical symmetry of the unperturbed Hamiltonian to the axial symmetry and treating the projection of the dynamic electric field onto the symmetry axis "preferentially"—more accurately than two other components.

In Refs. 1–3 we considered electron or ion impact broadening of spectral lines of neutral hydrogen and therefore dealt with rectilinear trajectories of perturbers. In the present paper we develop a generalized theory of electron impact broadening of spectral lines of hydrogen-like ions and therefore deal with hyperbolic trajectories of perturbing electrons employed in the corresponding SST.[4,7–12] In Sec. 2 we present the gist of all the generalized theories focusing on what is actually more general in them when compared to the SST. In Sec. 3 we specify our analytical results for electron impact broadening of hydrogen-like lines. In Sec. 4 we show that our numerical results compare better with experiments than calculations in the framework of previous theories and finally we draw conclusions in Sec. 5.

2. OUTLINE OF THE GENERALIZED DYNAMIC BROADENING THEORIES

Let us first review the structure of the SST. In the SST one deals with a quantum system described by the Hamiltonian

$$H(t) = H_0 + V(t), \quad V(t) = -\mathbf{dE}(t), \tag{1}$$

where H_0 is a time-independent atomic Hamiltonian, $V(t)$ is an interaction with the electric microfield. The key part of the formula for the lineshape $I_{ab}(\Delta\omega)$ is the impact broadening operator Φ_{ab}. The latter is expressed through the scattering matrix S:

$$\Phi_{ab} = N \int_0^\infty dv W_M(v) v \int_0^\infty d\rho 2\pi\rho \{S_a S_b^* - 1\}_{ang.aver.}. \tag{2}$$

The scattering matrix is calculated via the Dyson *perturbation* expansion:

$$S = T\exp[-i\hbar^{-1}\int_{-\infty}^{+\infty} dt q^* V q], \quad q \equiv \exp[-i\hbar^{-1}H_0 t]. \tag{3}$$

So in the SST only the unperturbed Hamiltonian H_0 is accounted for exactly, by its diagonalization in some coordinate system (usually—in the spherical quantization), while the entire interaction $V(t)$ is treated as a perturbation.

The central idea of the generalized theories[1-3] is to break down the interaction $V(t)$ in two parts

$$V(t) = V_1(t) + V_2(t) \tag{4}$$

and to rearrange the terms in Eq. (1) in such a way, i.e.,

$$H(t) = H_1(t) + V_2(t), \quad H_1(t) = H_0 + V_1(t), \tag{5}$$

that the extended "unperturbed" Hamiltonian $H_1(t)$ can be diagonalized at any instant of time (at least, in a subspace of a fixed principal quantum number n). Then the following modified Dyson expansion is used for the calculation of the scattering matrix:

$$S = \exp\left[-i\hbar^{-1}\int_{-\infty}^{+\infty} dt V_1(t)\right] T\exp\left[i\hbar^{-1}\int_{-\infty}^{+\infty} dt Q^* V_2 Q\right],$$

$$Q \equiv \exp\left[-i\hbar^{-1}\int_{-\infty}^{t} dt' H_1(t')\right] = \exp\left[-i\hbar^{-1}\left(H_0 t + \int_{-\infty}^{t} dt' V_1(t')\right)\right]. \tag{6}$$

Equation (6), which can be technically obtained by rearranging terms of the standard Dyson expansion allows one to take into account the interaction $V_1(t)$ on equal footing with the atomic Hamiltonian H_0, beyond the perturbation expansion.

Physically the split of the spherically symmetric interaction $V(t)$ into two axially symmetric parts $V_1(t)$, $V_2(t)$ and the preferential treatment of $V_1(t)$ requires a justification: the atomic Hamiltonian H_0 should have a symmetry lower than the spherical symmetry (e.g., the axial symmetry). Let us enumerate some situations where the spherical symmetry may get lowered to the axial symmetry

(1) H_0 contains an interaction with a static (or quasistatic) electric field. We studied this situation in Ref. 1.
(2) H_0 contains an interaction with a static (or quasistatic) magnetic field. We studied this situation in Ref. 2.
(3) H_0 is characterized by a motional anisotropy. We studied this situation in Ref. 3.

Now let us focus on common features of the generalized theories and answer why they are more general than the corresponding SST. First, the generalized theories may be characterized by the following controlling parameter

$$Y \equiv \langle H_{\text{anis}}\rangle/\Omega_W(V_1), \tag{7}$$

where H_{anis} is the anisotropic part of the atomic Hamiltonian $H_0 = H_{\text{isot}} + H_{\text{anis}}$; $\Omega_W(V_1)$ is the Weisskopf frequency characterizing the dynamic interaction $V_1(t)$. (Specifically for the generalized theory of electron broadening at the presence of the quasistatic electric field this parameter Y is given by Eq. (18) below or by Eq. (35) of Ref. 1.) The important point is that in the limit $Y \to 0$ we recover the corresponding SST. In other words, the generalized theories *embrace* the corresponding SST as the limiting cases.

Second, all the generalized theories are *convergent* at small impact parameters while the corresponding SST for neutral radiators were divergent. Physically the difference from the SST may be explained as follows. The generalized theories deal with virtual transitions, caused by the interaction $V_1(t)$, between atomic sublevels "*dressed*" by the interaction $V_2(t)$. It is the allowance for this "dressing" that eliminates the divergence and enhances the accuracy of the results.

Third, the scattering matrix, Eq. (6), (and consequently, the impact broadening operator) consists of two physically different terms:

$$S = S_a + S_{na}. \tag{8}$$

The first term

$$S_a \equiv \exp\left[-i\hbar^{-1}\int_{-\infty}^{+\infty} dt V_1(t)\right] \tag{9}$$

represents a purely adiabatic contribution, similar to what is usually referred as the "Old Adiabatic Theory" of broadening (see e.g. Ref. 6). As a rule, the main contribution to the broadening originates from the second term S_{na}

$$S_{na} \equiv \exp\left[-i\hbar^{-1}\int_{-\infty}^{+\infty} dt V_1\right]\left\{T\exp\left[i\hbar^{-1}\int_{-\infty}^{+\infty} dt Q^* V_2 Q\right]-1\right\}, \tag{10}$$

where after the expansion of $T\exp[\cdots]$ and the angular averaging, the first nonvanishing term is usually of the second order with respect to V_2. The term S_{na} is essentially non-adiabatic: it would vanish if there were not non-adiabatic virtual transitions caused by the interaction $V_2(t)$ between sublevels dressed by the interaction $V_1(t)$.

Fourth, it turns out that the generalized theories can be developed *analytically* to the same level as the corresponding SST. This is unexpected because the starting formulas for the generalized theories are more complicated than for the SST.

3. ANALYTICAL RESULTS FOR ELECTRON BROADENING OF HYDROGEN-LIKE LINES

We represent the Hamiltonian of a hydrogen-like ion (radiator) under the influence of an ion-produced quasistatic field **F** and an electron-produced dynamic field **E**(t) in the form of Eq. (5), with

$$H_0 = H_a - \mathbf{dF}, \quad V_1(t) = -d_z E_z(t), \quad V_2(t) = -\mathbf{d}_\perp \mathbf{E}_\perp(t) \equiv -d_x E_x - d_y E_y, \tag{11}$$

where H_a is the Hamiltonian of an isolated radiator, the axis z of the parabolic quantization is chosen along the field **F**. Then the truncated Hamiltonian $H_1(t) = H_0 + V_1(t)$ is diagonal in any n-subspace.

The detailed starting formulas are described by Eqs. (2)–(19) of our previous paper[1] and we will not reproduce them here. Then, in distinction to Ref. 1, we now use a hyperbolic trajectory of a perturbing electron with the usual parametrization:[4,7]

$$\mathbf{r}(\tau) = \frac{\rho}{K}\{f_v(\tau)\cdot\mathbf{e}_v + f_\rho(\tau)\cdot\mathbf{e}_\rho\}$$

$$r(\tau) = \rho\cdot(K\cosh(\tau)-\alpha)$$

$$f_y(\tau) = \alpha(K-\alpha\cosh(\tau)) + \sinh(\tau)$$

$$f_\rho(\tau) = K - \alpha(\cosh(\tau)+\sinh(\tau))$$

$$\mathbf{E}(\tau) = -\frac{e\mathbf{r}(\tau)}{r^3(\tau)}$$

$$\alpha = \frac{(Z_r-1)e^2}{\rho}\Big/(mv^2) = \frac{\rho_c}{\rho}$$

$$K = \sqrt{1+\alpha^2}$$

$$t = \frac{\rho}{V}(K\sinh(\tau)-\alpha\tau). \tag{12}$$

First we calculate the non-adiabatic contribution Φ_{na}. It turns out to be feasible, just as in our previous paper[1] to perform all three angular integrations analytically and to arrive at the

generalized broadening function $C(\chi, Y, \xi, Z)$ expressed in terms of elementary functions:

$$C(\chi, Y, \xi, Z) = \frac{3}{4K^2}\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1} dx_2 \frac{\exp\{iZ(K(\sinh x_1 - \sinh x_2) - \alpha(x_1 - x_2)\}}{((K\cosh x_1 - \alpha)(K\cosh x_2 - \alpha))^2}$$

$$\times \left\{ f_{\rho 1} f_{\rho 2} \cdot j_0(\epsilon) + (2 f_{v1} f_{v2} - f_{\rho 1} f_{\rho 2}) \cdot \frac{j_1(\epsilon)}{\epsilon} \right.$$

$$- ((f_{v1} f_{v2} - f_{\rho 1} f_{\rho 2}) \cdot \eta_1(x_1, x_2) + (f_{v1} f_{\rho 2} + f_{\rho 1} f_{v2})$$

$$\left. \times \eta_2(x_1, x_2)) \cdot \eta_2(x_1, x_2) \frac{j_2(\epsilon)}{\epsilon^2} \right\}$$

$$\epsilon = \sqrt{\eta_1(x_1, x_2)^2 + \eta_2(x_1, x_2)^2}$$

$$\eta_1(x_1, x_2) = -\frac{Y}{Z}\frac{1}{K}\left\{ \Theta_1(\tanh(x_2/2)) - \Theta_1(\tanh(x_1/2)) + 2\frac{\chi}{K} \right\}$$

$$\eta_2(x_1, x_2) = \frac{Y}{Z}\frac{1}{K}\left\{ \Theta_2(\tanh(x_2/2)) - \Theta_2(\tanh(x_1/2)) + 2\frac{\alpha\chi}{K} \right\}$$

$$\Theta_1(u) = \frac{2u + \frac{\alpha}{K}(1 - u^2)}{(K + \alpha)u^2 + (K - \alpha)}; \quad \Theta_2(u) = \frac{2\alpha u - \frac{1}{K}(1 - u^2)}{(K + \alpha)u^2 + (K - \alpha)}$$

$$\alpha \equiv \frac{\xi}{Z}; \quad K = \sqrt{1 + \alpha^2}. \tag{13}$$

Note that spherical Bessel functions $j_0(\epsilon)$, $j_1(\epsilon)$, $j_2(\epsilon)$ in (13) are indeed elementary functions:

$$j_0(\epsilon) = \epsilon^{-1}\sin\epsilon, \quad j_1(\epsilon) = \epsilon^{-2}(\sin\epsilon - \epsilon\cos\epsilon),$$

$$j_2(\epsilon) = \epsilon^{-3}(3\sin\epsilon - 3\epsilon\cos\epsilon - \epsilon^2\sin\epsilon). \tag{14}$$

Parameters Z and ξ are standard notations of the SST. The first one is defined as

$$Z_k \equiv \rho/(\rho_s)_k, \quad (\rho_s)_k \equiv \hbar v/[(\delta d)_k F], \quad k = \alpha, \beta;$$

$$(\delta d_\alpha) \equiv (d_z)_{\alpha\alpha} - (d_z)_{\alpha'\alpha'}, \quad (\delta d_\beta) \equiv (d_z)_{\beta\beta} - (d_z)_{\beta'\beta'}. \tag{15}$$

The second parameter is expressed as

$$\xi_k \equiv \rho_c/(\rho_s)_k, \quad \rho_c \equiv (Z_r - 1)e^2/(mv^2), \quad k = \alpha, \beta. \tag{16}$$

Compared to the SST, there are two new parameters that enter the generalized broadening function (just as in Ref. 1). The first one χ stands for

$$\chi_\alpha = [\mathrm{sign}(\delta d_\alpha)]X_{\alpha\beta}/n \quad \text{or} \quad \chi_\beta = [\mathrm{sign}(\delta d_\beta)]X_{\alpha\beta}/n', \quad X_{\alpha\beta} \equiv nq - n'q'. \tag{17}$$

The second new parameter Y is expressed as

$$Y_k \equiv [3n_k\hbar/(2Z_r m_e v)]^2 F/e \equiv \rho_W/\rho_s, \quad k = \alpha, \beta. \tag{18}$$

It is the controlling parameter previously described in Eq. (7) that provides a connection between our results and the SST. It can be shown that in the limiting case $Y \to 0$ corresponding to low- and

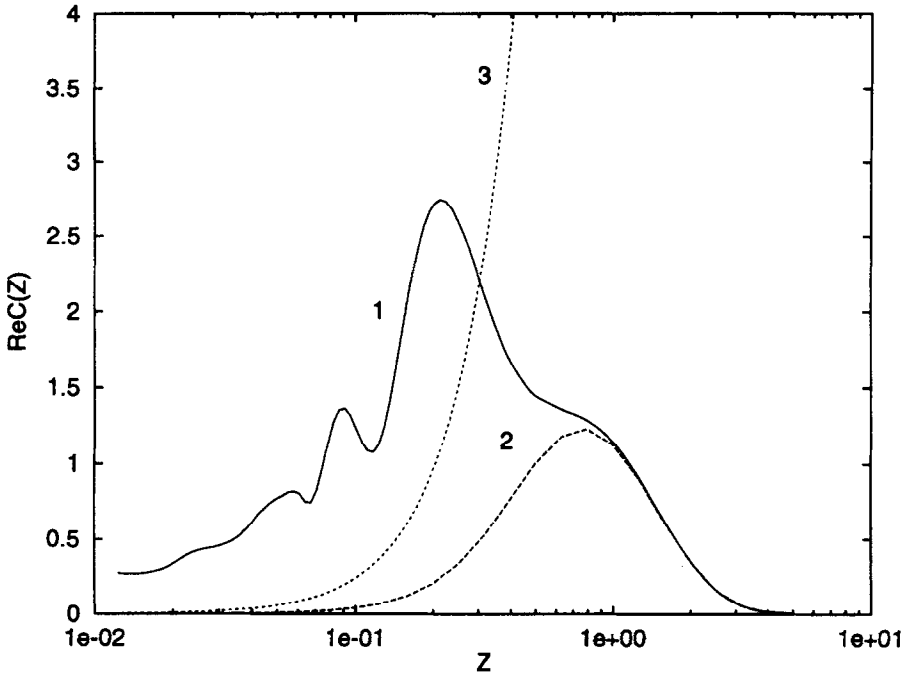

Fig. 1. The real part of broadening functions C calculated for $n = 2$, $Z_r = 2$, $\chi = 0$, $\xi = 0.37$, and $Y = 0.5$. The abscissa scale is logarithmic with respect to the reduced impact parameter Z, so that the non-adiabatic width γ_{na} [see Eq. (20)] is proportional to the area under the curve Re C. 1, Our theory, Eq. (13); 2, the SST, Eq. (19); 3, a unitarity-caused cutoff, Eq. (26) (shown only for our theory).

medium-density plasmas, from the generalized broadening function, Eq. (13), we recover the broadening function of the SST:†

$$C(\chi, 0, \xi, Z) = \frac{1}{2}\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{x_1} dx_2 \frac{\exp\{iZ(K(\sinh x_1 - \sinh x_2) - \alpha(x_1 - x_2)\}}{((K\cosh x_1 - \alpha)(K\cosh x_2 - \alpha))^2}$$

$$\times \{K^2 + \sinh x_1 \sinh x_2 + \alpha^2 \cosh x_1 \cosh x_2 - \alpha K(\cosh x_1 + \cosh x_2)\}. \quad (19)$$

However, the higher the density, the greater this parameter Y and the inaccuracy of the SST becomes.

A final expression for diagonal matrix elements of the non-adiabatic term of the impact broadening operator has the form:

$$(\Phi_{na})_{\alpha\beta} = -4\pi\hbar^2 N_e/(3m_e^2 a_0^2) \int_0^{\infty} dv W_M(v) v^{-1} \Bigg[\sum_{\alpha'} (|x_{\alpha\alpha'}|^2 + |y_{\alpha\alpha'}|^2)$$

$$\times \int_0^{\infty} dZ_\alpha Z_\alpha^{-1} C(\chi_\alpha, Y_\alpha, \xi_\alpha, Z_\alpha) + \sum_{\beta'} (\alpha \to \beta, \alpha' \to \beta')^* \Bigg];$$

$$W_M(v) \equiv 4\pi^{-1/2} v^2 v_0^{-3} \exp(-v^2/v_0^2), \quad v_0 \equiv (2T_e/m_e)^{1/2}. \quad (20)$$

Matrix elements of x and y operators in Eq. (20) differ from zero only for transitions between adjacent (by energy) Stark sublevels—due to peculiar selection rules for the parabolic quantization.

That is why each sum (over α' and over β') reduces to two terms: in the first term the integrand is $C(\chi_\alpha, Y_\alpha, \xi_\alpha, Z_\alpha)$, in the second term the integrand is $C(-\chi_\alpha, Y_\alpha, \xi_\alpha, -Z_\alpha)$.

Now we proceed to the calculation of the adiabatic contribution Φ_a. Following the steps described by Eqs. (46)–(50) of our previous paper,[1] we arrive to the same Eq. (51) of Ref. 1 for

†Speaking about this limit we emphasize again that it is not a part of the dynamic interaction $V(t)$ with the electron field that goes to zero. What goes to zero is the anisotropic part $H_{anis} = -\mathbf{dF}$ of the time-independent atomic Hamiltonian.

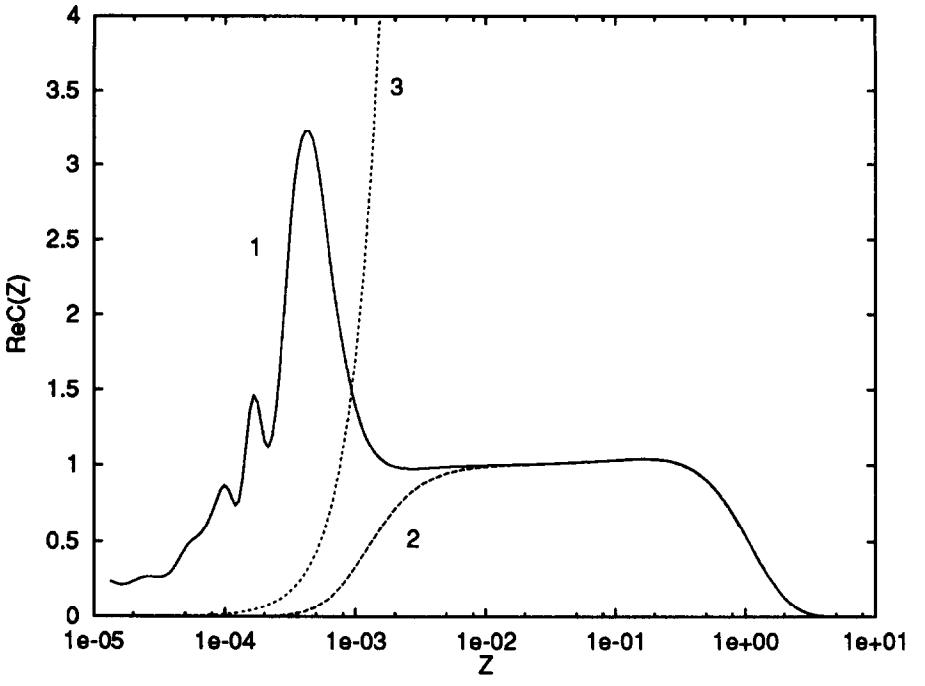

Fig. 2. The same as in Fig. 2, but for $n = 3$, $Z_2 = 2$, $\chi = 0$, $\xi = 8.51 \times 10^{-4}$ and $Y = 9.33 \times 10^{-4}$, where ξ and Y are calculated for the electron density $N_e = 9 \times 10^{17}\,\mathrm{cm}^{-3}$ and temperature $T_e = 7.5 \times 10^3\,\mathrm{K}$ by Eq. (25) with the substitution $F = 8.8eN_i^{2/3}$. The abscissa scale and the labels of the curves are the same as in Fig. 1.

the so-called "collision volume" with the only difference that $I(R)$ in Eq. (51) should be substituted by $I(R_1, R_2)$ where

$$I(R_1, R_2) = \int_{R_1}^{R_2} dz z(1 - z \sin z^{-1}) = G(R_1^{-1}) - G(R_2^{-1}),$$

$$G(x) \equiv [x^{-2}(\cos x - 3) + (2x^{-3} - x^{-1})\sin x + ci(x)]/6,$$

$$R_1 \equiv \rho_D/\rho_{Wa}, \quad R_2 \equiv \rho_c/\rho_{Wa}, \quad \rho_{Wa} \equiv 3X_{\alpha\beta}\hbar/(Z_r m_e v). \tag{21}$$

Here $ci(x)$ is the cosine integral function, ρ_D is the electron Debye radius. For the practically important range of $R_1 \gg 1$ it is sufficient to use an accurate asymptotic expression:

$$I(R_1, R_2) \approx [\ln R_1 + 1.26 + R_2^2(\cos R_2^{-1} - 3) + (2R_2^3 - R_2)\sin R_2^{-1} + ciR_2^{-1}]/6. \tag{22}$$

Note that in the limit $\rho_c \to 0$ (and consequently, $R_2 \to 0$) we recover from Eqs. (21), (22), formulas (52), (53) of Ref. 1 for the neutral hydrogen case.†

A final expression for the adiabatic contribution to the width has the form

$$(\gamma_a)_{\alpha\beta} = -\mathrm{Re}(\Phi_a)_{\alpha\beta} = 2\pi N_e \int_0^\infty dv W_M(v) v [\rho_{Wa}(v)]^2 I[R_1(v), R_2(v)]. \tag{23}$$

Taking into account that the function $v^{-1}W_M(v)$ has its maximum at $v = (T_e/m_e)^{1/2} \equiv v_{-1}$ and that

$$\int_0^\infty dv W_M(v) v^{-1} = (2m_e)^{1/2}(\pi T_e)^{-1/2},$$

†We would like to emphasize that for the neutral hydrogen case we had obtained in Ref. 1 the adiabatic contribution Φ_a *convergent* at small impact parameters while in the Old Adiabatic Theory it was divergent.[6] Physically it was achieved here by allowing for the vector character of summation of contributions from individual perturbers (while performing the angular averaging). In the Old Adiabatic Theory a scalar summation of perturber contributions was used.

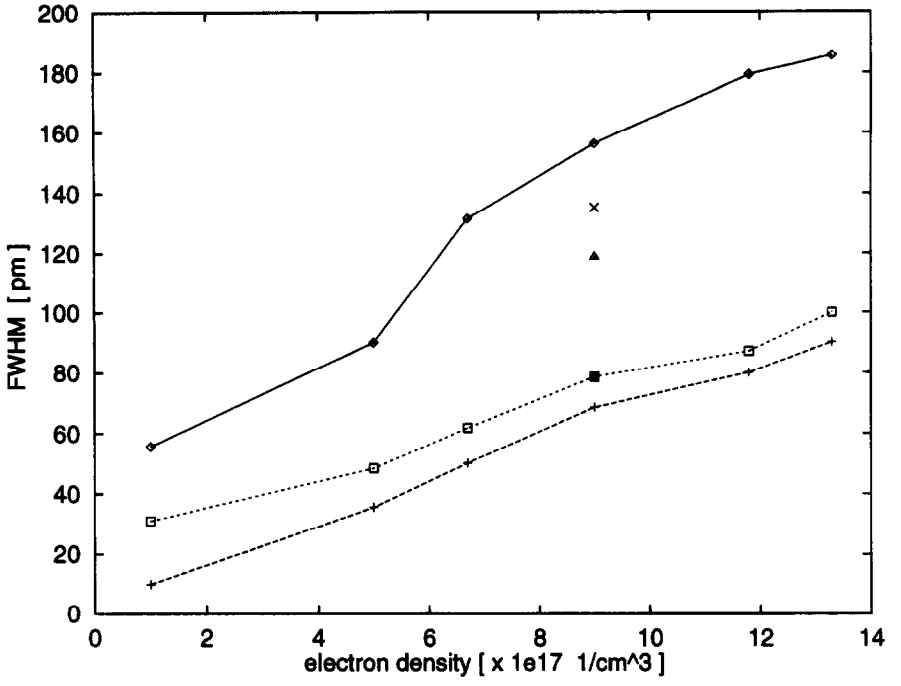

Fig. 3. Halfwidths of the H_α line of HeII vs the electron density. ◇, Experiment by Grützmacher and Johannsen[13] in a helium plasma of the temperature $T_e = 7.5 \times 10^3$ K; +, calculations by Günter et al[14] (Stark broadening only); □, calculations by Günter et al[14] (Stark + fine structure + Doppler broadening); *, calculations by Greene[15] (no ion dynamics); △, calculations by Greene[15] (ion dynamics included); ×, our calculations.

we can shortcut the integration over v by factoring the function $I[R_1(v_{-1}), R_2(v_{-1})]$ out of the integral in Eq. (23). Thus we arrive at the following expression for the total (non-adiabatic + adiabatic) impact width:

$$\gamma_{\alpha\beta} = 8\eta\left[\sum_{\alpha}(|x_{\alpha\alpha'}|^2 + |y_{\alpha\alpha'}|^2)\int_0^\infty dZ_\alpha Z_\alpha^{-1} C(\chi_\alpha, \langle Y_\alpha\rangle, \langle\xi_\alpha\rangle, Z_\alpha)\right.$$
$$\left. + \sum_{\beta'}(\alpha\to\beta, \alpha'\to\beta')^*\right] \Big/ 9 + 12\eta X_{\alpha\beta}^2 I(\langle R_1\rangle, \langle R_2\rangle),$$
$$\eta \equiv 3\pi\hbar^2 N_e (2Z_r^2 m_e^2)^{-1}(2m_e/\pi T_e)^{1/2}, \tag{24}$$

where

$$\langle Y_k\rangle = 9n_k^2 a_0 eF/(4Z_r^2 T), \quad \langle\xi_k\rangle = 3n_k e^2 F(Z_r - 1)(a_0/T_e^2)/(2Z_r),$$
$$\langle R_1\rangle = Z_r T_e/(3X_{\alpha\beta}\hbar\omega_{pe}), \quad \langle R_2\rangle = e^2 Z_r (Z_r - 1)(m_e/T_e)^{1/2}/(3X_{\alpha\beta}\hbar). \tag{25}$$

4. NUMERICAL RESULTS

In Fig. 1 we present the real part of our generalized broadening function, Eq. (13), calculated for $n = 2$, $Z_r = 2$, $\chi = 0$, $\xi = 0.37$, and $Y = 0.5$. The abscissa scale is logarithmic with respect to the reduced impact parameter Z, so that the non-adiabatic width γ_{na} [see Eq. (20)] is proportional to the area under curve 1 in Fig. 1. A comparison with the corresponding result of the SST (curve 2) shows that the SST significantly underestimates the electron impact broadening. Even after truncating the solid curve by curve 3 originating from the requirement of the unitarity of the S matrix ($|S_a - 1| \leqslant 2$), the remaining difference between the areas is about a factor of two.†

†Curve 3 is drawn in accordance to the formula below that follows from letting $|S_a - 1| = 1$:

$$\mathrm{Re}\, C_u(Z) = \{3Z_r^2/[(n^2 - q^2 - m^2 - 1)Y^2]\}Z^2. \tag{26}$$

In order to find out how our theory and the SST compare with experiments, we calculated the halfwidth of the H_α line of HeII for conditions of the experiment by Grützmacher and Johannsen.[13] In Fig. 2 we present the real part of our generalized broadening function, Eq. (13), calculated for $n = 3$, $Z_r = 2$, $\chi = 0$, $\xi = 8.51 \times 10^{-4}$, and $\langle Y_3(F)\rangle = \langle Y_3(8.8eN_i^{2/3})\rangle = 9.33 \times 10^{-4}$ at the electron density $N_e = 9 \times 10^{17}\,\mathrm{cm}^{-3}$ and temperature $T_e = 7.5 \times 10^3\,\mathrm{K}$ (all notations are the same as in Fig. 1). Again it is seen that the SST underestimates the electron impact broadening though not by a factor of two as in Fig. 1.

For the electron density $N_e = 9 \times 10^{17}\,\mathrm{cm}^{-3}$ and temperature $T_e = 7.5 \times 10^3\,\mathrm{K}$ previous calculations from Ref. 14 and from the "no-ion-dynamics" part of Ref. 15 ended up with halfwidths that were significantly (by a factor of two) smaller than the observed halfwidth of this line (see Fig. 3). Our generalized theory brings up the halfwidth closer to the experimental one by 20–25%. (The spread is caused primarily by the uncertainty of the unitarity cutoff.) If we would add up the dynamic ion contribution in accordance with Ref. 15, the discrepancy with the corresponding experimental result would be significantly reduced (see Fig. 3).

5. CONCLUSIONS

In the concluding discussion let us address some questions that may arise in conjunction with the generalized theories of the electron impact broadening. First, it might seem that in our previous paper[1] we claimed that for some plasma conditions the SST overestimates the broadening for neutral hydrogen while in the present paper we show that the SST underestimates the broadening for hydrogen-like ions. So the question might be asked: Why does it work in the opposite directions for hydrogen and for hydrogen-like ions? To begin with, in Ref. 1 we have shown that the most significant overestimation by the SST occurs for lateral components of the neutral hydrogen spectral lines; consequently, the most affected by that should be hydrogen lines without the central Stark components, like the H_β line. As for the central Stark components, for neutral hydrogen (i.e., $\xi = 0$) the situation strongly depends on plasma parameters, particularly on the controlling dimensionless parameter Y. It can be seen from Fig. 1 of Ref. 1 that the difference between the SST and the generalized, more accurate theory may be both ways: for some values of Y the SST overestimates the broadening while for some other values of Y the SST underestimates the broadening for neutral hydrogen. For hydrogen-like ions (i.e., $\xi \neq 0$) it seems that the SST practically always underestimate the broadening of the central Stark components to some extent, thus affecting the most significantly the hydrogen-like lines with intense central Stark components, like the H_α line.

Second, the question might arise concerning the central Stark components resulting from radiative transitions between the unshifted Stark sublevels $q = q' = 0$ ($q \equiv n_1 - n_2$ and $q' \equiv n_1' - n_2'$ are parabolic quantum numbers). Namely, since $\langle nq|d_z|nq\rangle = 0$ for $q = 0$, does it mean that the dressing by the interaction $V_1(t) = -d_z E_z(t)$ will be irrelevant for the central Stark components? The answer is: no. Indeed, the central Stark components are broadened due to virtual transitions between the unshifted and two adjacent Stark sublevels. However, the adjacent sublevels are dressed by $V_1(t)$. That is why Stark broadening of both the lateral and the central components is described in the generalized theories more accurately than in the SST. Note that the dressing become increasingly important for high-density plasmas.

Thus, both analytical and numerical results of the generalized theory of Stark broadening of hydrogen-like spectral lines presented in this paper demonstrate that the corresponding SST become increasingly inaccurate for high-density plasmas.

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