Changes in Fundamental Constants

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Abstract

This senior thesis explores the idea of varying fundamental constants. This variation can be over spacetime, or due to physical interactions. The connection between the values of the constants and life is explored via the Anthropic Principle and the idea of a multiverse. Further, current knowledge about the coupling constants is summarized. Deeper analysis is performed on the electromagnetic coupling constant, α, and how its variation would change atomic structure. Dark matter is introduced as a means of effective variation in the fundamental constants, according to the literature, and it is shown how some variations can be mathematically re-expressed to be consistent with current equations. Lastly, some modern evidence regarding variation is considered.

1 Introduction

Laws of physics are commonly expressed via equalities, which contain any number of variables and constants. Those values and their inter-dependencies are physically meaningful: they can be tested against the real world to be either observably true or untrue, and if they are shown to correspond to reality, then they give predictive power.

The complex web of physical relationships depends on constants. These constants in themselves have inter-dependencies, meaning that many of them can be or are derived from other constants. Following these equalities allows one to distill the numerous physical constants into a pool of fundamental constants, which are universal in importance, but cannot themselves be derived, and must be measured.

An example of such a constant is c, the speed of light. Historically, it was a variable that depended on the specific patch of light. This idea was encompassed by the theory of the luminiferous aether. However, numerous tests rejected the theory, and found that c did not change, neither by location nor even by the observer's velocity. Its constancy is the founding idea behind Special Relativity, and the value shows up in many other places, such as electromagnetism. This is what makes c a fundamental constant.

1.1 Physical Theories

There is no single definition of "fundamental constant", nor is there an agreed upon set of fundamental constants [1]. Indeed, what one considers to be a fundamental constant depends on the theories considered.

Certain constants are more well known, and more widely considered as fundamental. The fundamental constants of biggest fame are:

c: The speed of light (in vacuum), and the maximum speed of causality.

$\hbar$: The fundamental unit of spin, and the basis of quantum uncertainty.

G: The gravitational constant.

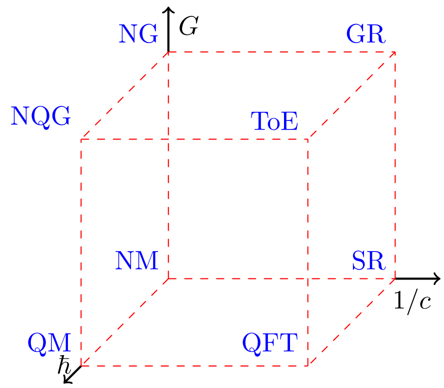


Figure 1: The Bronshtein-Zelmanov-Okun cube of physical theories: [1, 2],
NM = Newtonian Mechanics,
SR = Special Relativity, NG = Newtonian Gravity, QM = Quantum Mechanics,
GR = General Relativity, QFT = Quantum Field Theory, NQG = Nonrelativistic Quantum Gravity,
ToE = Theory of Everything

These three fundamental constants are highly important in modern physical theories. The inclusion or exclusion of these constants in theories creates a categorization scheme, known as the Bronshtein-Zelmanov-Okun cube, which is shown in Fig. 1 [1, 2]. It is important to note that the corners describe categories and not specific theories, though the naming of the categories typically follows the most well known theories.

Currently, the two best theories are General Relativity and the Standard Model (a Quantum Field Theory). The Standard Model includes over 20 fundamental constants [3], but it does not explain gravitation and does not include G.

Some of the fundamental constants used in the Standard Model include [1, 3]:

m_e: The mass of an electron.

m_p: The mass of a proton.

m_n: The mass of a neutron.

e: The magnitude of the charge of an electron.

Refer to appendix A.1 for a more inclusive list, as well as the accepted numerical values.

1.2 Naming Conventions

The constancy of the fundamental constants is an assumption, and it is an important question in physics as to whether their values vary or not. Furthermore, the "fundamental" aspect may be defined as the inability to be derived. Some sets of constants are inter-dependent, and knowing the values of some determines the values of others. To maintain some consistency, this will not immediately disqualify a constant from being fundamental. Thus, many fundamental constants will still be called such, despite not necessarily being fundamental or constant.

To quantify how many fundamental constants need to be measured to determine all relevant values, the values to be determined are called free parameters. The choice of which fundamental constants are considered free parameters is arbitrary, but the amount of free parameters is fixed within any given theoretical framework.

As an example, the fine-structure constant α_{EM} of Quantum Electrodynamics (QED) is defined as

$$\alpha_{EM} = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}, \tag{1}$$

where ϵ_0 is the electric constant, and the other constants are defined as before. Out of the five fundamental constants $\{\alpha_{EM}, e, \epsilon_0, c, \hbar\}$, only four may be considered free parameters, and the fifth can be derived from the other four.

The unitless value α_{EM} is a coupling constant, which determines the strength of electromagnetic interactions. The other forces have their own coupling constants as well. Because these constants are unitless, it is a major scientific quest to find a mathematical reason for their value. Doing so would reduce the amount of free parameters necessary to explain the universe.

Another big quest in physics is to find a Theory of Everything (ToE), which would combine the Standard Model with General Relativity. Such a theory could also reduce the number of free parameters. Some argue that $c, G, \hbar$ are the only true free parameters, from which all the other fundamental constants can be derived, and a ToE could provide the explanation, though currently no such theory exists [1, 2].

1.3 Relation to Measurements

The three fundamental constants $c, G, \hbar$ can be used to define the three main units of measurement: mass, distance and time. This is essentially a transformation of basis, as seen with

$$T(\{G, \hbar, c\}) = T_2(T_1(\{G, \hbar, c\})) = T_2(\{\,\mathrm{m_{Pl}}\,, \mathrm{l_{Pl}}\,, \mathrm{t_{Pl}}\,\}) = \{\,\mathrm{kg}\,, \mathrm{m}\,, \mathrm{s}\,\}. \tag{2}$$

The transformation T is defined by two separate transformations, T_1 and T_2. The transformation T_1 first converts the fundamental constants into Planck units, which are also known as natural units when the units are omitted (see appendix A.2 for their definition). The second transformation, T_2, then rescales the Planck units to output the more familiar SI units. T_2 can be replaced by some other transformation to yield other systems of measurement, or excluded entirely.

The set of fundamental constants used in modern theories has both unitless and unitful parameters. The unitful parameters present a challenge. Using the transformation T in equation (2) to define units of measurement inexorably intertwines them with the fundamental constants $G, \hbar, c$. Thus, if the value of a fundamental constant changes, the units with which we measure that constant could also change, and so the numerical values measured may be deceiving. In measuring the numerical values of $G, \hbar, c$, the result could simply echo the definition of the transformation T, in essence yielding a tautology. Using another means to define units of measurement only shifts the problem, and possibly obfuscates observations.

Thus, it may be only meaningful to measure unitless constants, and unitless ratios of constants, such as α_{EM} and $\mu = m_p/m_e$. It is possible to directly compare the values of a single constant in different spacetime regions, since this is a unitless ratio, but care has to be taken in order to avoid the above problem.

2 Philosophy

One of the questions concerning the fundamental constants is: Why do they have the values that they do? There is no known explanation, and so they are considered to be free parameters.

2.1 Anthropic Principle

One possible explanation for the values we observe is the Anthropic Principle [1], which links the presence of life to the fundamental constants. Any observations requires observers, and the presence of observers may necessitate specific physical laws and values of fundamental constants. Life cannot observe itself in an environment that does not support that life, therefore it is reasonable that we would find ourselves in a universe and locale that does support us. This can be used to explain fine-tuning – the fundamental constants are locked to specific values because we are here to observe those values.

For example, $1/\alpha_{EM} \approx 137 = Z_{\mathrm{crit}}$ is an approximate upper limit to the atomic number, since larger atomic numbers greatly favor inverse β-decay of electrons and nucleus protons, which reduces the atomic number and stabilizes the atom [4]. We can see that current observations are consistent with this limit; we know of no elements that have more than 137 protons. Variations of α_{EM} would lead to changes in

[1] There are two variation, the Weak and Strong Anthropic Principles. The strong version states that the universe *must* come to support life, wherein fine-tuning is an inherent requirement. The Weak Anthropic Principle does not hold such a requirement.

chemistry, and at $1/\alpha_{EM} \approx 1$, atomic structures become unstable, as is demonstrated later in this thesis for the hydrogen atom.

Other considerations can place even stricter bounds on potential values of various fundamental constants. Requiring that the lifetime of a proton exceeds the current age of the universe places an upper bound on α_{EM}, and requiring the presence of a Grand Unified Theory implies a lower bound [1]:

$$1/170 < \alpha_{EM} < 1/80 \tag{3}$$

2.2 Multiverse

The Anthropic Principle could explain why the fundamental constants hold the values that they do, but by restricting possible values, the odds of a universe bearing life becomes successively less probable. Theoretical calculations and simulations of the universe under a slightly changed set of fundamental constants show that the universe would be vastly different [5]. A small nudge would have large consequences, likely preventing life.

By using the Anthropic Principle, we have shifted the problem from finding out why the fundamental constants are the way they are, to finding out why life exists. After all, our universe could have had different parameters resulting in an absence of life, though consequently no one would be around to observe such. The probability of life seems remote, yet it exists.

One way to resolve this problem is via the idea of a multiverse. If there are many universes, each one with its own set of fundamental constants, then there are many opportunities for life to happen. An arbitrarily large amount of universes could essentially guarantee that life exists somewhere, and then that life would observe hospitable conditions.

To define what a multiverse is or could be, it's first necessary to define what a universe is. Unfortunately, there are several definitions. A possible definition is the set of all contiguous space that is causally linked two-ways.

If points A and B can influence each other, then they can be said to be in the same universe. This definition is useful, but fragile. For example, our universe is expanding, and some parts of it are receding beyond our causal range [6]. In this way, what a universe is, is time-dependent and observer-dependent.

Taking Earth to be point A, there are two points B and C, such that A and B are causally linked, and A and C are casually linked, but B and C are not. The lack of the transitive property is not intuitive in this definition.

Following this, some possible definitions of a multiverse are:

I: Continuous Multiverse: It is possible to imagine that, in a universe according to one observer, there are many other observers who see regions outside the first observer's universe. Each of those observers can in turn function the same as the first observer. Then, our universe can be continuously extended into a multiverse.

II: Discrete Multiverse: It is also possible that our observable universe lives inside a bubble, one of many others. These bubbles may exist separate from each other, perhaps inside a space of eternal inflation that serves as a separation medium [2].

III: Container Multiverse: Similar to the previous idea, it is possible that bubble universes contain each other. If they exist, black hole universes are contained universes, because it would be theoretically possible to enter them, but not to exit them. Container multiverses only have one-way causal linkage.

IV: Quantum Multiverse: Assuming the many-worlds interpretation of quantum mechanics, each collapse of a wave function spawns many worlds – the different quantum universes are expressions of the possibilities. In this way, our universe perpetually branches into an arbitrarily large amount of alternate universes.

V: Mathematical Multiverse: The broadest definition of the multiverse dictates that everything that mathematically could be, is. By this definition, all realities that are self-consistent exist.

If a multiverse allows for the fundamental constants to take on different values in different universes, then a fine-tuned universe is almost guaranteed.

Unfortunately, the more easily a multiverse could guarantee fine-tuning, the less testable and practical it is. On the ridiculous side, a mathematical multiverse guarantees everything that is self-consistent, but it is also completely untestable.

For the scientist, a multiverse presents a challenge of falsifiability. A universe that is entirely causally disconnected from our reality has no hope of being testable. A one-way causal linkage allows for some testing, but not for the whole scientific process – the tester either has no control of the input of the experiment, or they cannot obtain the output.

This renders the idea of a multiverse scientifically dubious. One possibility of testing a multiverse is to use better definitions. It may happen that, as our knowledge expands, what we see as causally connected expands as well – in other words, our scientific reach may expand beyond what we have previously seen as our universe.

3 Coupling Constants

The Standard Model states that there are four fundamental forces: the electromagnetic, weak, strong, and gravitational forces [2]. All interactions are mediated by one or more of these four, and the strengths of these forces are determined by unitless coupling constants $\{\alpha_f\}$, where f is a stand-in for the respective force. A bare α is the usual shorthand for α_{EM}, and α is also known as the fine-structure constant.

Interestingly, we already know that the coupling constants $\{\alpha_f\}$ change, and their dynamic values are expressed as $\alpha_f(E)$, where E is the energy of the interaction [3], and $\alpha_f(E)$ is a function of that energy.

This property is described by quantum dynamics. In Quantum Electrodynamics, the true charge [4] of an electron is hidden by the polarized vacuum around it, and this screening of the charge lowers the effective charge of the electron [3]. At higher energies, particles can come closer to each other, and the screening effect decreases.

The change to α_{EM} can be linked to the change of e, as in:

$$\alpha'_{EM} = \frac{e'^2}{4\pi\epsilon_0\hbar c}, \tag{4}$$

where α'_{EM} and e' are the modified EM coupling constant and modified fundamental electric charge, respectively. The other fundamental constants do not change in this model.

For electromagnetism, increased interaction energies increase the effective electron charge, and thus increase the effective coupling constant. A similar effect happens to the strong and weak forces, except their respective coupling constants decrease at higher energies.

When interaction energies surpass the rest energy of the $W^\pm$ and Z bosons of the weak force (as calculated by $E = mc^2$ from rest mass m), which occurs past $90\,\mathrm{GeV}$, the electromagnetic and weak forces can be modelled as a single force [3], the electroweak force. In this energy range, $\alpha_{EM} \approx 1/128$ [7]. These energies are achievable in current colliders, which provide positive evidence for this unified force. In this model, α_{EM} and α_W are not fundamental constants in themselves, nor independent – the true fundamental constant is α_{EW}, and the other two coupling constants may be derived from it. The reason that the electromagnetic and weak forces appear as separate forces has to do with spontaneous symmetry breaking, wherein an outcome of a symmetric law need not be symmetric.

This unification may be taken one step forward to include the strong force, and a theory describing the unification of the three fundamental forces would be called a Grand Unified Theory (GUT). Unfortunately, such a unification is predicted to happen at energies above $\sim 10^{15}\,\mathrm{GeV}$ [3, 8], which are not currently achievable. Furthermore, most potential GUTs have issues, such as a finite half-life for protons. While the predicted lifetimes for protons in such models are around $10^{30}\,\mathrm{yr}$, which greatly surpasses the current age of the universe, current observations place a lower bound that is above most GUT-predicted proton lifetimes.

[2] Though the gravitational force is not modelled by the Standard Model

[3] While E is usually not a relativistic quantity, it is possible to find the scale of the energy of an interaction between two particles by multiplying their four-momenta: $p_1 \cdot p_2$. Thus, the coupling constants can be described relativistically.

[4] The widely accepted electron charge $-e$ is taken to be the fully screened charge and not the "true charge".

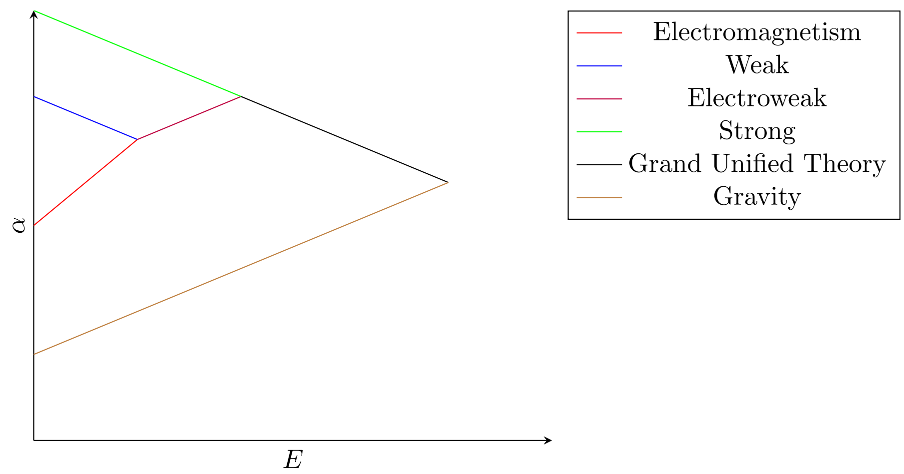


Figure 2: The potential trend of coupling constant values over a range of interaction energies, such that they become united at high enough energies.

However, if a GUT becomes widely accepted, the three coupling constants representing the three forces may be united, further reducing the number of free parameters. The final step after a GUT is to bring in gravity. In a Theory of Everything (ToE), all four forces may be described as a single force, with a single coupling constant, α_{ToE}. Figure 2 shows a rough sketch of how the four base coupling constants may merge into a single coupling constant at higher energies, if a ToE can unify the four forces. However, there is no guarantee that the coupling constants exhibit this trend. The creation of a ToE is a dramatic challenge, because unlike the other three forces, there is currently no relativistic quantum theory of gravity.

For the rest of this thesis, we will assume that the coupling constants $\{\alpha_f\}$ refers to the base values $\{\alpha_f(0)\}$ – the coupling constants as they are at zero energy – unless otherwise stated.

3.1 The EM Coupling Constant

Of the four fundamental forces, electromagnetism is most understood. Its coupling constant is measured to extreme precision, and has led to some of the best predictions in quantum mechanics [9, 3]. The best current theory is Quantum Electrodynamics.

In QED, Feynman diagrams are visual representations that describe possible interactions between real particles, and those diagrams encode mathematical expressions that indicate the likelihood of those interactions. For any given set of real particles, there is an infinite set of potential interactions mediated by virtual particles. The overall diagram can be described by a combination of interconnected vertices, and each additional vertex modifies the probability of the associated interaction roughly by α [3]. Hence, by controlling the probabilities, α dictates the strength of interactions. Feynman diagrams apply to Quantum Flavordynamics (QFD) and Quantum Colordynamics (QCD) as well, though the precise rules are slightly different.

3.2 Schrödinger Equation and Hydrogen

Suppose we desire to calculate the effect of varying α on atomic structure. For simplicity, we will work with hydrogen-like atoms, which contain Z protons in the nucleus, and one bound electron.

While QED offers great precision, it is difficult to calculate for even this simple case. There are several alternatives in quantum dynamics that may be easier to work with, but whose results may not be as accurate as QED.

Schrödinger's (time-dependent) equation is [3]

$$\underbrace{\left(-\frac{\hbar^2\nabla^2}{2m} + V\right)}_{H}\Psi = i\hbar\partial_t\Psi, \tag{5}$$

where Ψ is the scalar wave function of a particle, m is the mass of that particle, $\hbar$ is defined as before, ∇^2 is the Laplacian operator, V is potential energy, H is the Hamiltonian operator, and both Ψ and V may vary over space and time. This is a simple, non-relativistic model.

For a Coulomb potential, with a single electron orbital around a point charge of magnitude Ze, the potential is

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} = -\frac{Z\alpha\hbar c}{r}, \tag{6}$$

where r is the distance between the single electron and the point nucleus. For such a potential, the resulting binding energies of the orbitals are [10]

$$E_n = -\frac{m_e c^2 Z^2 \alpha^2}{2n^2} \approx -13.6\frac{Z^2}{n^2}\,\mathrm{eV}, \tag{7}$$

where n is the principal quantum number that specifies the orbital of the single electron, and we take $\alpha \approx 1/137$.

Using the Schrödinger equation to model a hydrogen atom (when $Z = 1$), with the currently accepted value of α, the first binding energy is

$$E_1 \approx -13.6\,\mathrm{eV}. \tag{8}$$

This result agrees with observed values, but this model excludes relativistic effects and ignores electron spin, so this model may not work well at different values of α.

3.3 Dirac Equation and Hydrogen

A single electron orbital may also be modelled by the Dirac equation [10],

$$\underbrace{\left(c\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta m_e c^2 + V\right)}_{H_D}\phi = E\phi, \tag{9}$$

where ϕ is the bi-spinor (spinor with four components) wave function describing the electron,

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \tag{10a}$$

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{10b}$$

are 4x4 matrices, $\boldsymbol{\sigma}$ are the three Pauli spin matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{11a}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{11b}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{11c}$$

$\boldsymbol{p} = -i\hbar\nabla$ is the three-momentum operator defined via the gradient, V is the potential energy as before, H_D is the 4x4 matrix Dirac Hamiltonian, E is the total energy of the particle, and c and m_e are defined as before. This equation is occasionally expressed in atomic units (see appendix A.3 for an explanation).

This Dirac equation is both relativistic and accounts for the electron's spin. However, the inclusion of a potential V ignores some of the details of QED, and as such, some precision is lost. For example, the Lamb shift cannot be predicted by this model. Furthermore, the potential V is a Coulomb potential, and thus has a singularity at $r = 0$, which skews the analysis.

For this model, the energy levels are given by [10]

$$E_{n,\kappa} = m_e c^2 \Big/ \sqrt{1 + \frac{\alpha^2 Z^2}{(\gamma + n - |\kappa|)^2}}, \tag{12}$$

where $E_{n,\kappa}$ is the total energy of the electron, n, c and Z are defined as before,

$$\gamma = \sqrt{\kappa^2 - \alpha^2 Z^2}, \tag{13a}$$

$$\kappa = \mp(j + 1/2) \text{ when } j = l \pm 1/2, \tag{13b}$$

j is the total angular momentum, and l is the orbital angular momentum, and all angular momentum excludes units of $\hbar$. Because $E_{n,\kappa}$ is the total energy, the binding energy must be calculated by subtracting the rest energy $m_e c^2$ from $E_{n,\kappa}$.

Note that the energy levels depend on $|\kappa|$, and thus on the total angular momentum j, but not directly on the orbital angular momentum l – there is degeneracy here that is lifted by the Lamb Shift not included in this model. While an electron orbital may be described by the quantum numbers n and l (and m_l, the z-component of the angular momentum), it is the overall hydrogen-like atomic system that dictates the electron's energy levels in this model. For any given n, $l = 0, 1, ..., n$, and $j = l \pm 1/2 > 0$. This implies that $\kappa = -n, -n+1, ..., -1, 1, ..., n-1$. These bounds dictate the possible values of the quantum numbers, for which the wave function is defined and well behaved.

If we plug in values for a hydrogen atom ($Z = 1$) and assume the currently accepted value of α, we get that the first binding energy is

$$E_{1,-1} - m_e c^2 \approx -13.6\,\text{eV}, \tag{14}$$

which matches both the Schrödinger model and observations.

This model is demonstrably correct, and should still hold under different values of α, which lets us hypothesize how a hydrogen atom would behave under varying α. In Figure 3, we plot the first few orbital energy levels. The results are:

$\alpha = 0$: the electromagnetic force has no strength and ceases to exist, which can be seen by the electron's energy level equalling its rest energy for all orbitals.

$0 < \alpha < 1$: all the normal orbitals exist meaningfully. As α increases and approaches 1, the magnitude of the binding energy of all orbitals increases. The closer α approaches to 1, the more relativistic the binding energy becomes, especially for the $n = 1$ ($1s$) orbitals.

$\alpha = 1$: the orbitals represented by $n = 1$ neither add nor subtract energy to the nucleus-electron system, which implies that such a bound electron can pop into and out of existence spontaneously (ignoring other conservation laws). All $n = 2$ orbitals exist at this point, but the solutions to the equation governing the energy of the $2s_{1/2}, 2p_{1/2}$ orbitals border the complex solutions, and those are assumed to be non-physical. This may either indicate a physical property of the system, or a problem with the model.

$1 < \alpha < 2$: only $2p_{3/2}$ (for which $n = 2$, $\kappa = -2$, $j = 3/2$, $l = 1$) is physical, and it continues the trend of deepening the binding energy as α increases.

$\alpha = 2$ the $2p_{3/2}$ orbital is at the zero energy level, and ceases to exist meaningfully after this point within this model.

The existence of the orbitals can be analyzed by observing the behavior of $\gamma = \sqrt{\kappa^2 - \alpha^2 Z^2}$; the value under the square root must be non-negative for the orbital to be real and stable, otherwise complex values of γ yield complex energies, which do not represent stable orbitals. We have real values of γ only if

$$\kappa^2 \geq \alpha^2 Z^2 \Rightarrow \alpha \leq |\kappa|/Z. \tag{15}$$

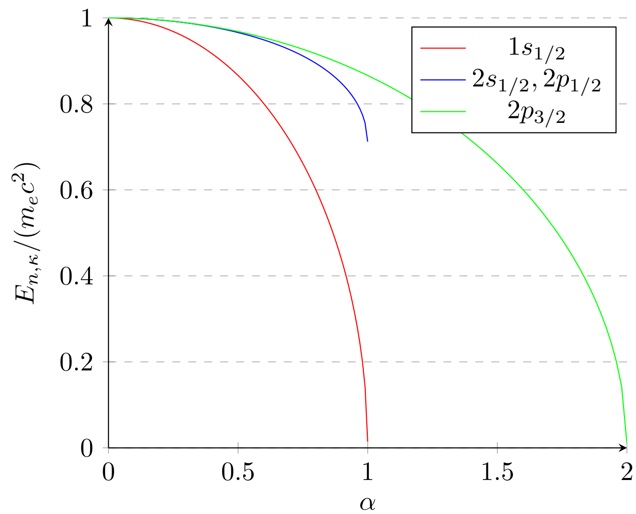


Figure 3: Energy levels of a bound electron around a hydrogen ($Z = 1$) point-like nucleus.

This implies that the value of α puts a lower bound on $|\kappa|$ such that orbitals are still meaningful, and any given value of $|\kappa|$ yields physically meaningful orbitals for values of α up to the critical threshold

$$\alpha_{\text{crit}} = |\kappa|/Z = (j + 1/2)/Z = (|l \pm 1/2| + 1/2)/Z. \tag{16}$$

This can be interpreted to mean that a strong enough electromagnetic force requires orbital electrons to maintain a higher orbital angular momentum l in order for the orbital to be stable. Because $l < n$, some principal quantum numbers become forbidden at high enough values of α.

3.4 Non-Point-Like Corrections

One of the problems of the above analysis is that we take the nucleus to be point-like. The resulting potential V has a singularity at the origin that becomes relevant at high enough values of the product $Z\alpha$. In order to extend our analysis of single-electron orbitals into negative energies, we must use another potential.

To avoid the singularity, we can model the nucleus as a spherical shell with radius [11]

$$R \approx 1.2 A^{1/3}\,\text{fm}, \tag{17}$$

where A is the mass number. This results in a truncated Coulomb potential

$$V_{tr}(r) = \begin{cases} -\frac{Z\alpha\hbar c}{r}, & r > R \\ -\frac{Z\alpha\hbar c}{R}, & r < R \end{cases}, \tag{18}$$

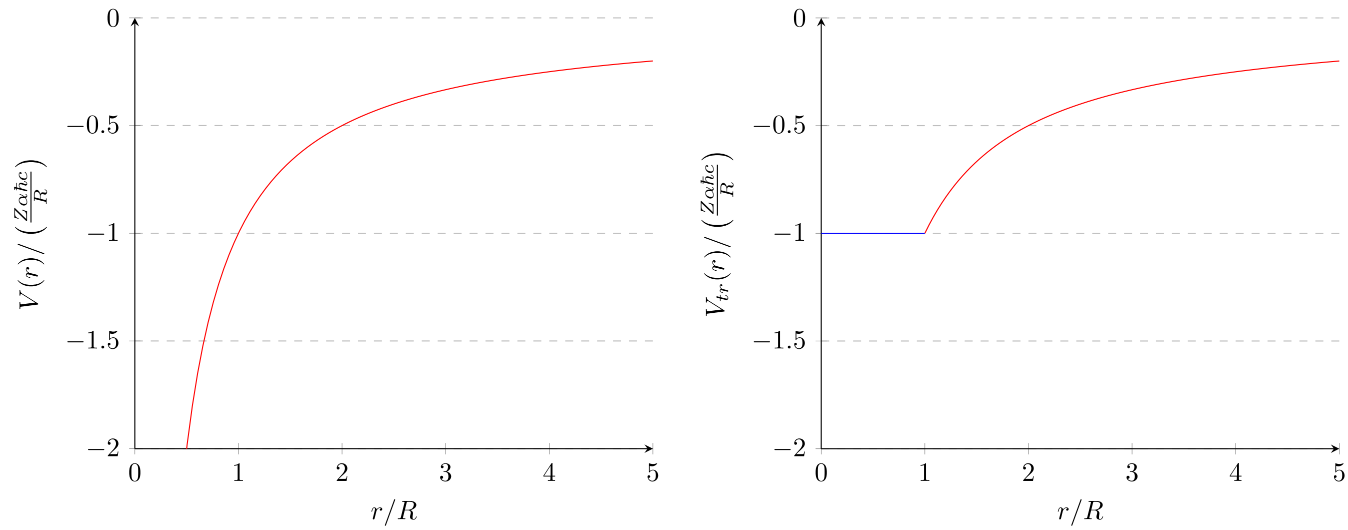
where r is the distance from the center of the nucleus, and all other parameters are defined as before.

A comparison between the Coulomb potential and its truncated version is shown in Fig. 4. Both Fig. 4a and Fig. 4b are rescaled by the radius R in order to be comparable.

While a shell nucleus is not a fully accurate model, it serves as a better model than the point-like nucleus, and simplifies some calculations while avoiding the nasty singularity.

Because our new model should allow us to analyze the behavior of an atom with negative energy orbitals, we may performs calculations for when the $1s_{1/2}$ orbitals dip into $E = -m_e c^2$. This is our new condition for criticality, because it is at this point that the vacuum destabilizes and produces electron-positron pairs [11].

A nucleus with an orbital whose energy dips below $-m_e c^2$ must have such an orbital filled, otherwise it is spontaneously filled by the production of bound electrons, with associated unbound positrons produced from the excess energy to satisfy charge conservation laws. Any attempts to ionize that orbital would rather produce more electron-positron pairs. In this way, the nucleus and the electron orbital stop being separate objects.



(a) Coulomb potential $V(r)$ around a point-like nucleus.

(b) Truncated Coulomb potential $V_{tr}(r)$ around a shell nucleus.

Figure 4: Two different potentials around a nucleus.

Our focus will be directed on the $1s_{1/2}$ orbital, because it reaches criticality first. Other orbitals have their own criticality points, and they happen at more extreme values of α and Z.

Solving the Dirac equation at the critical energy $E = -m_e c^2$ for the $1s_{1/2}$ orbital yields [11]:

$$\frac{QK'_{i\nu}(Q)}{K_{i\nu}(Q)} = 2(Z\alpha)\cot(Z\alpha), \tag{19}$$

where K is the modified Bessel function of the second kind, its derivative is in respect to Q, and

$$\nu = 2\sqrt{(Z\alpha)^2 - \kappa^2}, \tag{20a}$$

$$Q = \sqrt{\frac{8m_e c^2 Z\alpha R}{\hbar c}}. \tag{20b}$$

The necessary computations were carried out with the help of the Wolfram Notebook. The code and output can be seen in appendix B, including some graphs.

For $Z = 1$,

$$\alpha_{\text{crit}} \approx 1.1333. \tag{21}$$

That is to say, for a hydrogen atom, the $1s_{1/2}$ orbital energy dips below $-m_e c^2$ for $\alpha > \alpha_{\text{crit}} \approx 1.1333$.

Given a value of α, we also calculate the maximum Z for which the $1s_{1/2}$ orbital energy remains above $-m_e c^2$, rounded down. For $\alpha = 1/137$,

$$Z_{\text{crit}} \approx 173, \tag{22}$$

which agrees with literature values [11]. For $\alpha = 1$,

$$Z_{\text{crit}} \approx 1. \tag{23}$$

At currently accepted values of $\alpha \approx 1/137$, nuclei may have as many as 173 protons before the EM field gains the ability to spontaneously produce real electron-positron pairs. However, at $\alpha = 1$, only the hydrogen nucleus is able to exist without spontaneously polarizing the vacuum.

Roughly,

$$Z_{\text{crit}} \propto 1/\alpha, \tag{24a}$$
$$\alpha_{\text{crit}} \propto 1/Z. \tag{24b}$$

The reason for this is because, in our criticality equation (19), most times Z and α are together, in the form of the product $Z\alpha$. If this were always the case, then a specific value of $Z\alpha$ would solve the equation, from which the above relations would hold. The exception occurs in our estimation of the nuclear radius R, which depends on the atomic mass, which further depends only on Z in our model, but not α.

Recall that in our previous analysis, using the Dirac equation and modelling the nucleus as a point, we calculated α_{crit} for $E = 0$ energy levels, and there, $\alpha_{\text{crit}} \propto 1/Z$ as well. This correspondence reinforces both analyses.

3.5 Summary of Varying α

Based on this work, varying α changes atomic structure, and limits permissible atomic numbers that do not polarize the vacuum.

The value of α plays a huge role in chemistry. A different value of α could fundamentally change what atoms are stable, or prevent atomic structures altogether. Even at $\alpha = 1/70$, which is around twice the current accepted value, uranium ($Z = 92$) ceases to function as it does now, and its atomic structure degrades. By knowing the stability of atoms, we can put limitations on the potential values of α.

It is important to note that these analyses apply to bare nuclei, with single bound electrons, and that they ignore the effects of α on the structure of the nucleus – since protons, and even the quarks that make up protons and neutrons, are charged, interactions inside the nucleus could be impacted by a varying α.

4 Effective Variation

There may be any number of reasons why the value of a fundamental constant varies. Some searches are concerned about variation across spacetime, wherein the base fundamental constant changes for no apparent reason. However, there can also be interactions that effectively modify the value. Those interactions are made implicit by including them in the effective value of a fundamental constant, which is then used in most other places.

4.1 Dark Matter

Dark matter may induce such interactions. If dark matter interacts in some ways with particles of the Standard Model, it is possible for this interaction to locally change the effective values of the fundamental constants. For example, the masses of fermions and the coupling constants may effectively vary, as described by [12, 13]

$$m_{f,\text{eff}} = m_f \left[1 + \Gamma_f^{(n)} \left(\phi\sqrt{\hbar c}\right)^n\right], \tag{25a}$$
$$\alpha_{\text{eff}} \approx \alpha \left[1 + \Gamma_\alpha^{(n)} \left(\phi\sqrt{\hbar c}\right)^n\right], \tag{25b}$$

where $m_{f,\text{eff}}$ and α_{eff} are the effective fermion mass and coupling constant values, m_f and α are the fermion mass and coupling constant values before the dark matter perturbation, $n = 1, 2$ is the order of the dark matter Lagrangian, the $\{\Gamma\}$ are coupling constants between the dark matter and fermion/boson field, and ϕ is the dark matter field itself.

This variation can be detected by a network of sensors, such as atomic clocks positioned around the Earth [12, 13]. Such sensors may be used to detect the variation of fundamental constants, but they may also indicate a new physical interaction or phenomenon, such as dark matter.

4.2 Refactoring the EM Force

The variation of the fundamental constants introduces a new problem: Their derivatives are no longer identically zero. This can play a large role in writing physical equations, but it may be possible to use clever mathematics to reformat old equations to fit with such variations.

Suppose that the effective electric charge varies:

$$e_{\text{eff}} = e\epsilon, \tag{26}$$

where e_{eff} is the effective/apparent fundamental electric charge, ϵ is some perturbation around 1, and e is the base fundamental electric charge. This variation can be linked to a variation in α via equation (1).

This variation demands us to rewrite the electromagnetic tensor $F_{\mu\nu}$ as [2] [5]

$$F_{\mu\nu} = \frac{\partial_\mu(\epsilon A_\nu) - \partial_\nu(\epsilon A_\mu)}{\epsilon}, \tag{27}$$

where A_μ is the electromagnetic four-potential. For non-varying ϵ, the regular form of the electromagnetic tensor is recovered.

The above can be simplified by defining auxiliary variables,

$$f_{\mu\nu} = \epsilon F_{\mu\nu}, \tag{28a}$$
$$a_\mu = \epsilon A_\mu, \tag{28b}$$

to yield the expression

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu. \tag{29}$$

Thus, we are back at the familiar form of the EM force, and the variation of the electric charge has been magicked away as part of the force. Both formulations are mathematically equivalent, but there is a conceptual difference. Care has to be taken because the physical meaning of $f_{\mu\nu}$ and a_μ may not match with that of $F_{\mu\nu}$ and A_μ, though only the EM fields $F_{\mu\nu}$ (or $f_{\mu\nu}$) in particular are directly observable.

5 Evidence of Variation

There have been many experiments attempting to measure potential variation in the fundamental constants. One of the more well known experiments involve the natural nuclear reactor in Oklo, Gabon. A varying α would impact nuclear rates, and these rates can be compared against current fission content in the reactor. The resulting approximate bound in the variation of α is [1]

$$\frac{\dot{\alpha}}{\alpha} = (-0.8 \pm 5.9) \times 10^{-17}\,\text{yr}^{-1}\,, \tag{30}$$

with a 2σ confidence level. Different analyses of the nuclear reactor give different bounds, depending on the assumptions built into the analyses.

Another experiment compares Al^+ and Hg^+ optical clocks, which yields a unitless ratio that provides a similar bound on the variation of α [14]:

$$\frac{\dot{\alpha}}{\alpha} = (-1.6 \pm 2.3) \times 10^{-17}\,\text{yr}^{-1}\,. \tag{31}$$

Combination of many optical clock experiments and other experiments yields some of the best bounds [14]:

$$\frac{\dot{\alpha}}{\alpha} = (-0.7 \pm 2.1) \times 10^{-17}\,\text{yr}^{-1}\,, \tag{32a}$$
$$\frac{\dot{\mu}}{\mu} = (0.2 \pm 1.1) \times 10^{-16}\,\text{yr}^{-1}\,, \tag{32b}$$

[5] Here, we assume the flat spacetime of Special Relativity. General Relativity makes a distinction between the normal partial derivative δ_μ, and the absolute covariant derivative Δ_μ, which would complicate our analysis.

where $\mu = m_p/m_e$ as before.

Various stellar observations about the age of stars constrain the probable variation of the gravitational constant [1]:

$$\frac{\dot{G}}{G} = (-0.6 \pm 4.2) \times 10^{-12}\,\mathrm{yr}^{-1} \tag{33}$$

All of these observations are consistent with zero variation, meaning that it is still considered to be a scientific possibility that the fundamental constants do not change over spacetime.

6 Summary

Physics relies on fundamental constants, which appear in many important theories that model reality. These fundamental constants may change over spacetime or due to interactions. The Anthropic Principle may explain why the fundamental constants have the values that they do, and a multiverse may provide an explanation for the likelihood of fine-tuning. The coupling constants are important to the stability of various structures in our universe. This senior thesis explains the effects that a varying EM coupling constant would have, using various quantum mechanical models. There is a rough inverse relationship between the EM coupling constant and atomic numbers corresponding to stable atomic structures. Dark matter interactions are a possible explanation for a variation in the EM coupling constant, as well as fermion masses. Various observations and experiments attempted to measure possible variations in several fundamental constants, and currently, there is insufficient conclusive evidence that the fundamental constants vary over spacetime.

References

[1] Jean-Philippe Uzan. The fundamental constants and their variation: observational and theoretical status. *Rev. Mod. Phys.*, 75:403–455, Apr 2003.

[2] Mariusz P. Dabrowski. Anthropic selection of physical constants, quantum entanglement, and the multiverse falsifiability. *Universe*, 5(7), 2019.

[3] David J Griffiths. *Introduction to elementary particles; 2nd rev. version.* Physics textbook. Wiley, New York, NY, 2008.

[4] George Greenstein and Allen Kropf. Cognizable worlds: The anthropic principle and the fundamental constants of nature. *American Journal of Physics*, 57(8):746–749, 1989.

[5] Don Lincoln. 33 subatomic stories: Does the multiverse exist? `https://www.youtube.com/watch?v=Uu5zvQzEhaI`, Nov 2020.

[6] Don Lincoln. How far is the edge of the universe? `https://www.youtube.com/watch?v=u23vZsJbrjE`, Oct 2019.

[7] Introduction to the constants for nonexperts. `https://physics.nist.gov/cuu/Constants/alpha.html`.

[8] Ugo Amaldi, Wim de Boer, and Hermann Fürstenau. Comparison of grand unified theories with electroweak and strong coupling constants measured at lep. *Phys. lett. B*, 260(IEKP-KA-91-01):447–455, 1991.

[9] Matt O'Dowd and Graeme Gossel. Quantum theory's most incredible prediction |space time. `https://www.youtube.com/watch?v=7UwigY4SjKY`, Aug 2018.

[10] Walter R. Johnson. *Lectures on Atomic Physics.* Physics textbook. University of Notre Dame, Notre Dame, Indiana 46556, jun 2006.

[11] W. Greiner and J. Reinhardt. *Quantum Electrodynamics.* Springer Berlin Heidelberg, 2013.

[12] Andrei Derevianko. Atomic clocks and dark-matter signatures. *Journal of Physics: Conference Series*, 723:012043, jun 2016.

[13] Andrei Derevianko and Szymon Pustelny. Global quantum sensor networks as probes of dark sector. Sep 2020.

[14] M. S. Safronova, D. Budker, D. DeMille, Derek F. Jackson Kimball, A. Derevianko, and Charles W. Clark. Search for new physics with atoms and molecules. *Reviews of Modern Physics*, 90(2), Jun 2018.

Appendices

A Constants and Units

A.1 Constants

Defined in SI units $\mathrm{kg}, \mathrm{m}, \mathrm{s}$ and other derived units, the constants used throughout this thesis and their accepted values are:

- Speed of light: $c \approx 2.998 \times 10^8\,\mathrm{m\ s^{-1}}$
- Reduced Planck constant: $\hbar \approx 1.055 \times 10^{-34}\,\mathrm{J\ s} \approx 6.582 \times 10^{-22}\,\mathrm{MeV\ s}$
- Gravitational constant: $G \approx 6.673 \times 10^{-11}\,\mathrm{m^3\ kg^{-1}\ s^2}$
- Mass of electron: $m_e \approx 0.5220\,\mathrm{MeV\ c^2}$
- Mass of proton: $m_p \approx 938.3\,\mathrm{MeV\ c^2}$
- Mass of neutron: $m_n \approx 939.6\,\mathrm{MeV\ c^2}$
- Mass $W^{\pm}$: $m_W \approx 80.42\,\mathrm{MeV\ c^2}$
- Mass of Z: $m_Z \approx 91.19\,\mathrm{MeV\ c^2}$
- Positive magnitude of electron charge: $e \approx 1.602 \times 10^{-19}\,\mathrm{C}$
- Electromagnetic coupling constant: $\alpha_{EM} \approx \frac{1}{137.0359895} \approx 0.007297353080$

A.2 Planck Units

Planck units are defined in terms of $\hbar, \mathrm{G}, \mathrm{c}$ (and sometimes k_B).

Setting

$$\hbar = G = c = k_B = 1 \tag{34}$$

defines natural units.

Conversion between Planck and SI units:

- $\mathrm{l_{Pl}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35}\,\mathrm{m}$
- $\mathrm{t_{Pl}} = \mathrm{l_{Pl}}/c = \sqrt{\frac{G\hbar}{c^5}} \approx 5.391 \times 10^{-44}\,\mathrm{s}$
- $\mathrm{m_{Pl}} = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8}\,\mathrm{kg}$
- $\mathrm{E_{Pl}} = \mathrm{m_{Pl}}c^2 = \sqrt{\frac{\hbar c^5}{G}} \approx 1.956 \times 10^9\,\mathrm{J}$
- $\mathrm{T_{Pl}} = \frac{\mathrm{E_{Pl}}}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}} \approx 1.417 \times 10^{32}\,\mathrm{K}$

A.3 Atomic Units

Atomic units are defined [10, 14] by declaring that

$$m_e = \frac{e}{\sqrt{4\pi\epsilon_0}} = \hbar = 1. \tag{35}$$

In these units,

- $c = 1/\alpha \approx 137.0359895$
- Rest energy of electron: $E_{e,0} = m_e c^2 = c^2 = 1/\alpha^2 \approx 18778.86241$

B Wolfram Notebook Program

Below is the PDF copy of my notebook used to calculate critical values: α_{crit} versus Z, and Z_{crit} versus α.

To rid myself of units, I used

$$8 * \frac{(1.2\,\text{fm})m_e c^2}{\hbar c} \approx 0.02486. \tag{36}$$

I inputted specific atomic masses of small nuclei, and used the estimation

$$A \approx (1.45 + 0.24\ln(Z))\,Z, \tag{37}$$

to obtain approximate masses of larger nuclei. Here, A is the expected atomic mass number of stable isotopes. This is a homebrewed estimate, which is arrived at by assuming the general form, then solving for two points, $Z = 10$ (neon) and $Z = 80$ (mercury). This formula yields values within ± 2 of most stable isotopes, and for large nuclei, this margin of error does not significantly impact the estimate of R, nor the resulting values of α_{crit} and Z_{crit}.

The discrete plots are plotted with

```
DiscretePlot[y-axis-array, x-axis-range]
```

which is important to know for interpreting the title-less graphs produced. For the second graph,

$$\text{ai} = 1/\alpha \tag{38}$$

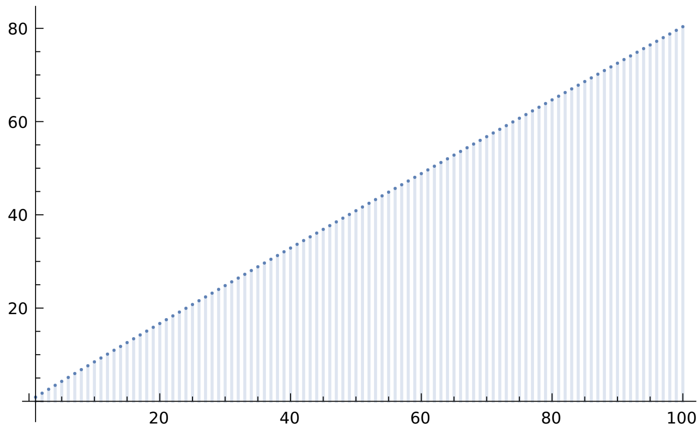
is used for convenience.

Later, I test that R indeed plays a role, both graphically and by testing the value of $Z\alpha$. While the last two graphs are similar, R serves to slightly rescale one of the expressions, which happens when Z varies, but not when α varies.

```
In[1]:= Clear["Global`*"];

In[2]:= masses = {1, 4, 7, 9, 11, 12, 14, 16, 19, 20};
        mass[Z_] := If[Z < Length[masses], masses[[Floor[Z]]], (1.45 + 0.24 * Log[Z]) * Z];

In[4]:= Za := Z * a;
        R := (mass[Z])^(1/3);
        Q := Sqrt[0.02486 * Za * R];

In[7]:= k = -1;
        i = Sqrt[-1];
        iv := i * 2 * Sqrt[Za^2 - k^2];

In[10]:= Kiv := BesselK[iv, Q];
         dKiv := iv/Q * BesselK[iv, Q] - BesselK[iv + 1, Q];

In[12]:= leftEq := Re[Q * dKiv / Kiv];
         rightEq := 2 * Za * Cot[Za];

In[14]:= aCrit := FindRoot[leftEq == rightEq, {a, 1/Z}][[1, 2]];

In[15]:= as = Range[1, 100];
         For[Z = 1, Z ≤ 100, Z++, as[[Z]] = aCrit]
         as[[1]]
         1/as

Out[17]= 1.13336

Out[18]= {0.882333, 1.73416, 2.5806, 3.42785, 4.27154, 5.11883, 5.95725, 6.79348, 7.62083,
  8.46006, 9.28913, 10.1166, 10.9424, 11.7669, 12.59, 13.4119, 14.2326, 15.0521,
  15.8706, 16.6881, 17.5046, 18.3202, 19.1348, 19.9486, 20.7616, 21.5738,
  22.3852, 23.1958, 24.0058, 24.815, 25.6235, 26.4314, 27.2387, 28.0453,
  28.8513, 29.6567, 30.4615, 31.2658, 32.0695, 32.8726, 33.6753, 34.4774,
  35.279, 36.0802, 36.8808, 37.6809, 38.4806, 39.2799, 40.0787, 40.877, 41.6749,
  42.4724, 43.2695, 44.0661, 44.8624, 45.6582, 46.4537, 47.2487, 48.0434,
  48.8377, 49.6317, 50.4252, 51.2185, 52.0113, 52.8039, 53.596, 54.3879,
  55.1794, 55.9705, 56.7614, 57.5519, 58.3421, 59.132, 59.9216, 60.7108,
  61.4998, 62.2885, 63.0769, 63.8649, 64.6527, 65.4402, 66.2274, 67.0144,
  67.8011, 68.5874, 69.3736, 70.1594, 70.945, 71.7303, 72.5154, 73.3002,
  74.0847, 74.869, 75.6531, 76.4369, 77.2204, 78.0037, 78.7868, 79.5696, 80.3522}
```

In[19]:=
```
DiscretePlot[1/as[[Z]], {Z, 1, 100}]
```

Out[19]=



In[20]:=
```
ZCrit := FindRoot[leftEq == rightEq, {Z, 1/a}][[1, 2]];
```

In[21]:=
```
a = 1/137;
ZCrit
```

Out[22]= 173.066

In[23]:=
```
Zs = Range[1, 140];
For[ai = 1, ai ≤ 140, ai++, a = 1/ai; Zs[[ai]] = ZCrit];
Zs
```

Out[25]=
```
{1.13336, 2.30659, 3.48756, 4.66765, 5.85269, 7.05023, 8.2432, 9.44779, 10.651,
 11.859, 13.0698, 14.283, 15.4986, 16.7165, 17.9363, 19.1582, 20.3819, 21.6073,
 22.8344, 24.0632, 25.2934, 26.5252, 27.7583, 28.9929, 30.2288, 31.4659, 32.7043,
 33.9439, 35.1846, 36.4265, 37.6695, 38.9135, 40.1586, 41.4047, 42.6518, 43.8999,
 45.1489, 46.3989, 47.6498, 48.9015, 50.1541, 51.4076, 52.6619, 53.917, 55.1729,
 56.4296, 57.6871, 58.9454, 60.2044, 61.4641, 62.7245, 63.9857, 65.2476, 66.5101,
 67.7733, 69.0373, 70.3018, 71.567, 72.8329, 74.0994, 75.3665, 76.6342, 77.9025,
 79.1714, 80.4409, 81.711, 82.9817, 84.253, 85.5248, 86.7971, 88.07, 89.3435,
 90.6175, 91.892, 93.167, 94.4426, 95.7187, 96.9952, 98.2723, 99.5499, 100.828,
 102.107, 103.386, 104.665, 105.945, 107.226, 108.506, 109.788, 111.07, 112.352,
 113.635, 114.918, 116.202, 117.486, 118.77, 120.055, 121.34, 122.626, 123.912,
 125.199, 126.486, 127.773, 129.061, 130.349, 131.638, 132.927, 134.216,
 135.506, 136.796, 138.087, 139.378, 140.669, 141.961, 143.253, 144.545,
 145.838, 147.131, 148.425, 149.718, 151.013, 152.307, 153.602, 154.897,
 156.193, 157.489, 158.785, 160.082, 161.379, 162.676, 163.974, 165.272,
 166.57, 167.869, 169.168, 170.467, 171.767, 173.066, 174.367, 175.667, 176.968}
```

In[26]:=
```
Zs[[1]]
Zs[[137]]
Zs[[70]]
```

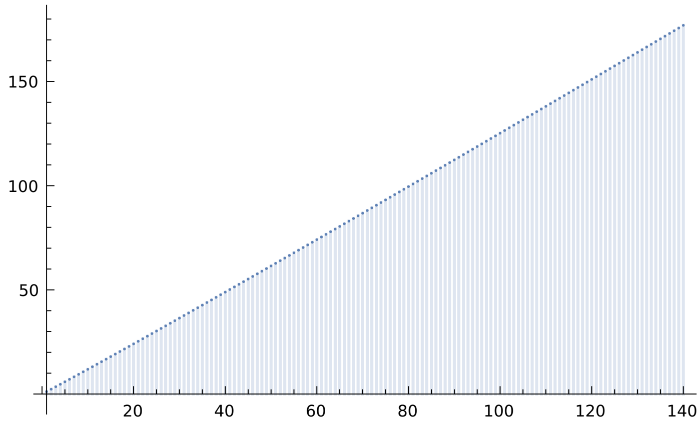
Out[26]= 1.13336

Out[27]= 173.066

Out[28]= 86.7971

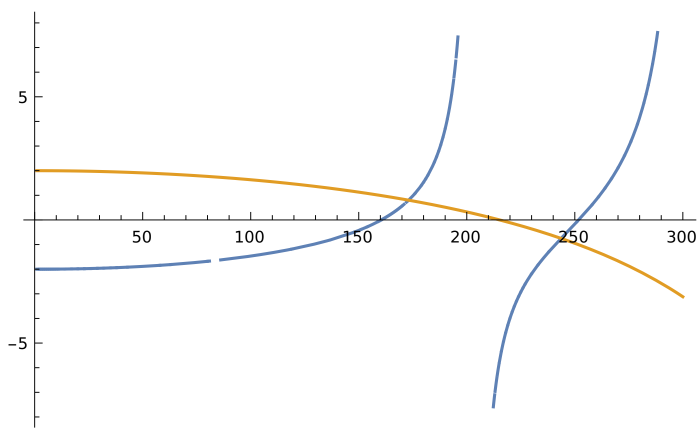
In[29]:=
```
DiscretePlot[Zs[[ai]], {ai, 1, 140}]
```

Out[29]=



In[30]:=
```
a = 1 / 137;
Plot[{leftEq, rightEq}, {Z, 1, 300}]
a * ZCrit
```

Out[31]=

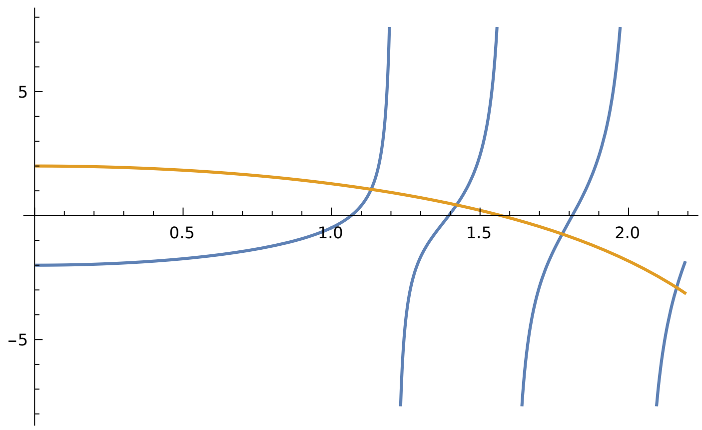


Out[32]= 1.26326

In[33]:=
```
Z = 1
Plot[{leftEq , rightEq}, {a, 1 / 137, 300 / 137}]
Z * aCrit
```

Out[33]= 1

Out[34]=



Out[35]= 1.13336